

THE APPLICATION OF
LINEAR ECONOMIC MODELS TO MARKETING

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INTRODUCTION

The conception of the term "marketing" is usually attributed to Ralph Starr Butler, who, in 1910, published six printed pamphlets titled Marketing Methods which were later published as a textbook titled Marketing. Butler states, "In brief, the subject matter that I intended to treat was to include a study of everything that the promoter of a product has to do prior to his actual use of salesmen and of advertising."¹ In order to examine the above subject matter, marketing scholars have developed the institutional, functional, and commodity approaches for the formulation of marketing principles.

The application of these approaches to marketing problems has generated a wealth of information. However, the results of these approaches have not been integrated into any logical frame of reference so as to form the foundations for the statement of marketing principles. In the last decade, considerable research has been performed in an attempt to accomplish such an integration. The domain of marketing thought is no longer confined to Butler's interest in the promotion of products, but has expanded to a multi-dimensional discipline.

To date, the major part of the research done in the field of marketing belongs to the structural dimension, i.e., an examination of the basic components of the marketing system. The commodity,

¹ R. Bartels, The Development of Marketing Thought (Homewood, Ill.: R. D. Irwin, 1962), p. 225.

institutional, and functional approaches to marketing provide an analysis of the structural dimension of marketing. However, such an analysis does not consider the influence of the temporal dimension on marketing phenomena. In addition to neglecting the temporal dimension, traditional market analysis also fails to emphasize the effects of geographical location, the spatial dimension, in its explanation of the market mechanism. Until recent years the advances made in the other social sciences were not used to any large extent in the field of marketing. A combination of this interdisciplinary dimension of marketing with the previous dimensions will provide a basis for the development of marketing principles, the intellectual dimension of marketing.

In recent years, many marketing scholars have attempted to synthesize this multi-dimensional approach to marketing so as to provide a conceptual frame of reference for the solution of marketing problems. A leading exponent of this approach is Professor W. Alderson whose functionalism is an integration of the commodity, functional, and institutional approaches.² In addition to integrating these traditional approaches, Alderson also increases their scope. He emphasizes the structural relationships between marketing units and insists that such relationships are dynamic in contrast to the traditional static approaches to marketing.

Functionalism is a recent innovation in marketing theory, consequently, it is not stated in a rigorous fashion. Although

²W. Alderson, Marketing Behavior and Executive Action (Homewood, Ill.: R. D. Irwin, 1957).

Alderson's functionalism has not been defined by a mathematical model, conceptually, it has emphasized the structural interdependence of marketing activities. However, in order that functionalism may be used in the solution of marketing problems, it must be stated in quantitative terms. As one critic states, "In sum, the challenge facing the student of marketing is that of operationalizing the conceptual schemata suggested by functionalism--that is, of translating them into empirical research instruments."³

The object of this thesis is to use a mathematical model, which exemplifies the concept of functionalism, for an examination of current marketing problems. The model to be used is an economic model developed by Professor W. Leontief. The model, which is usually called an input-output model, was chosen for a variety of reasons. First, the traditional approaches to marketing may be used for collecting and organizing in a logical and consistent manner the enormous amount of data required for the use of an input-output model.

Second, Alderson's functionalism emphasizes the structural relations between marketing units. The interdependency of firms is inherent in an input-output model. Another property of functionalism is that it offers a dynamic interpretation of marketing activity as opposed to the static theory represented by the institutional, commodity, and functional approaches to marketing. Given specific conditions, the Leontief model can be used for a dynamic analysis of

³F. M. Nicosia, "Marketing and Alderson's Functionalism," Journal of Business, XXV (1962), 412.

marketing activities. The remainder of this thesis involves the application of this linear economic model to the functionalist approach in marketing.

CHAPTER 1

THEORETICAL CONSIDERATIONS IN THE DEVELOPMENT OF MARKETING MODELS

1.1 The Need for Mathematical Models in Marketing

The need for mathematical models in marketing is an outgrowth of two recent marketing phenomena. First, there is a need for a theoretical framework which will provide a means for integrating the enormous amount of existing information concerning various facets of the market. Second, the scope of marketing analysis has increased as a result of its maturing as an academic discipline.

The Increase in Marketing Data

Consider the increase in marketing data. Since the development and use of the electronic computer, marketing data have increased exponentially. Not only is there an increase in the availability of national data but there is also a corresponding increase in the data for the individual firm in the market. With the introduction of electronic computers, firms now collect and store more data concerning their internal operations. The problems confronting the market analyst are to determine the best way to interpret and analyze these data. When the data are meager, the market analyst has only to consider a few variables before he renders a decision. However, when the data become quite detailed, the interrelationships between each set of data are not

obvious; therefore the analyst needs some conceptual frame of reference into which he can place the data.

A model provides the market analyst with such a conceptual frame of reference. For the purpose of this discussion, a model is defined as a set of symbolic relationships which abstracts some phase of the marketing process.

The connection between the use of models and the growth in empirical information which describes the marketing process is obvious. The use of a model will enable the market analyst to synthesize all of the relevant data in a concise way so that it will form a conceptual frame of reference. Without the use of a model, it would be impossible to analyze all the interdependencies between the different sets of data in many complex marketing systems.

For example, consider the problem of distributing a sales promotion budget over different regions. Prior to the recent advances in the collection of marketing data, such decisions could be made without the use of models since the available data upon which the decision depended were meager. However, the present day market executive has reams of information upon which he draws before rendering a decision. As the data become more detailed, the executive cannot make optimum use of all the information without the use of a model. For example, it is one problem to allocate advertising expenditures over the given sales regions by knowing only each region's total sales and profit; however, the problem becomes more difficult when additional data are known such as: the sales promotion activities of the competitors, income levels

of the average customer, employment trends, consumer preferences, the educational level of the consumers, the nature of the product mix, geographic sales distribution within the region, and customer sales by nature of employment. By using models, the market analyst is able to determine the quantitative effects of the change of one variable on the other variables in the system.

Marketing, A Maturing Discipline

The other need for models in marketing results from the new domain of marketing inquiry. To the surprise of many academicians, marketing is no longer limited to study of salesmanship, retailing, wholesaling, and sales promotion in a given firm. The market analyst is now concerned with consumer behavior beyond the level of an individual firm. He is a specialist whose primary interest is in the nature and functioning of the market, the most basic component of a capitalistic society.

In order to study the market, the market analyst must consider many interrelationships between place, product, promotion, price, government control, income, employment, and consumer demand. This list is by no means exhaustive; it merely illustrates the changing interests of those engaged in marketing.

One of the greatest advances in marketing methodology is the tendency towards a macro approach to marketing problems. Until recent years, the primary interests of those engaged in marketing involved the marketing activities of the individual firm, i.e., a micro approach was used in lieu of a macro approach. Studies on the micro level furnish

basic data for the construction of general concepts, which, when stated in mathematical form, become models. Therefore, the development of marketing concepts and marketing models is closely related; i.e., the latter is a symbolic statement of the former.

The use of models permits a holistic approach to the study of marketing in that they can handle the interrelationships between the many economic and social systems in the market. Such a synthesis of socio-economic systems was not needed on the micro level thereby restricting the use of mathematical models.

The following chapters contain an application of a linear economic model to a macro study of the market mechanism. However, it should not be inferred that the use of mathematical models in marketing is restricted to the macro level. The need for marketing models arising from an increase in the number of variables to be considered before rendering any market decision was discussed in the aforementioned advertising example. The number of variables to be considered can increase on the micro level as well as on the macro level as evidenced by the need for models in the previously discussed sales promotion problem.

The Advantages in Using Models in Marketing

In the majority of instances, the need for mathematical models in marketing is actually a necessity since certain problems could not be solved without the utilization of a model. This section contains a discussion of the advantages in using models in lieu of the traditional qualitative approaches to marketing. The implication of this section is that the market analyst should use models, when applicable in his

analysis, even though they are not an absolute necessity. Some of the advantages in using mathematical models in marketing are:

1. Synthesis-- A mathematical model synthesizes the relevant variables so that they may be subjected to a variety of statistical inferences.
2. Comprehension-- The use of models permits a more rigorous and clearer statement of marketing principles.
3. Explicitness-- In order to construct a model, the assumptions used in its formulation must be stated in symbolic form thereby making them explicit.
4. Description-- Mathematical models use symbolic notation which generally permits a more accurate description of marketing phenomena than those descriptions given by qualitative marketing concepts.
5. Computation-- A model is easily adapted to statistical techniques which may be programmed on an electronic computer thereby reducing the time involved for a numerical analysis.
6. Data Collection-- The model states explicitly the data needed for the analysis therefore eliminating the problem of collecting extraneous data.
7. Depth of Analysis-- The use of a model permits the use of advanced mathematical techniques which increase both the scope and depth of the analysis.
8. Extension-- Additions to or modifications of the basic model can be made quite easily in order that it may be made more comprehensive.

1.2 The Concept of Mathematical Marketing Models

The Role of Model Building in Marketing Analysis

The first step in the quantitative solution of marketing problems by the use of mathematical models is model building or specification. The other steps are: estimation, verification, and prediction. This chapter concentrates on model building, i.e., specification. Chapters 3,

4, 5, and 6 discuss and illustrate the techniques involved in the steps of estimation and prediction. Chapter 7 concentrates on the problem of verification.

In the previous section, a marketing model is defined as a set of symbolic relationships which abstracts some phase of the marketing process. The process of expressing market phenomena in terms of mathematics is called specification or model building. Inaccurate specification procedures lead to model error which is the most difficult to correct of the three major sources of error in model building. Chapter 7 contains a detailed analysis of the major sources of error in model building.

The problem of specification is quite complex in the construction of models in the social sciences, in particular, marketing models. Each marketing transaction represents a countless number of considerations on the part of the consumer. The successful model builder must choose the most pertinent variables out of this great number. This process is one of the most difficult problems in model building; it is the central problem in the specification of marketing models. "Successful model building requires an artist's touch, a sense of what to leave out if the set is to be kept manageable, elegant, and useful with the collected data that are available."¹ The problem of estimation, verification, and prediction are defined and illustrated later in this chapter.

¹S. Valavanis, Econometrics (New York; McGraw-Hill, 1959), p. 1.

Mathematical Models in the Natural Sciences

Since the physical scientist has been building mathematical models for many years, it is natural for the social scientist to investigate the methodology of the physical scientist and to use it, when applicable, for the construction of models in the social sciences.

The dispute concerning the validity in paraphrasing natural science models by the use of economic analogies is a lengthy one. Some economists state dogmatically that such paraphrasing is without a doubt an excellent approach to the development of economic models. "Some writers (economists) attempt to distinguish between statics and dynamics by analogy with what they understand to be the relationship in theoretical physics. That this is a fruitful and suggestive line of approach cannot be doubted."²

Other economists state that the use of natural science models in economics is useless.

Examples of situations where mechanical behavior models are applicable are: chemical systems, biophysical systems, and types of structure analyzed in classical physics. Unfortunately, however, the conditions under which the application of a mechanical type behavior model is most useful do not exist in these aspects of the world which are studied by economics. The attempt to describe its behavior by one of these mechanical models may be convenient and desirable for many purposes, but it does not make for successful prediction, or any kind of successful application.³

Logically the position of the author should be one of reconciliation. However, the author is in support of the latter argument

²P. A. Samuelson, Foundations of Economic Analysis (Cambridge, Mass.: Harvard University Press, 1947), p. 311.

³S. Schoeffler, The Failures of Economics: A Diagnostic Study (Cambridge, Mass.: Harvard University Press, 1955), p. 19.

subject to a few minor modifications, i.e., in the opinion of the author, paraphrasing natural science models leads to the development of erroneous marketing models. Since the current dispute is one of methodology, no attempt will be made to prove the above statement.

The author is well aware of the fact that it may be possible to paraphrase a natural science model and arrive with a meaningful marketing model. Although this has never been accomplished, it is absurd to state that it may be impossible to do so. However, in economics, the Leontief model has had more empirical uses than any other model and at no part of its derivation have analogies been drawn between natural science models and market phenomena. It was constructed from an examination of pertinent market forces; not a quasi-scientific approach based on forced analogies between economic and natural science phenomena. Similarly, those who state that without doubt, the models of the natural sciences will eventually lead to significant advances in marketing models are also astray. Although it is impossible to construct marketing analogies of all models in the natural sciences, the construction of a particular model in the natural sciences is discussed in order to give an example of the limitations of the methodology used in the construction of social science models. The model to be discussed is the well known ideal gas law. The selection of this model is not the result of a haphazard choice. The primary reason for the choice of this model is that its assumptions are common to many natural science models. Therefore many of the criticisms which apply to the ideal gas model also apply to natural science models in general.

Although the author has a prejudice before the analysis is undertaken, he attempts to describe those analogies between natural science concepts and marketing phenomena which are useful in addition to those which lead to the development of invalid marketing models.

A Natural Science Model.-- The ideal gas law is a natural science model which relates three important properties of any gas, namely its pressure, volume, and temperature. The problem of specification is quite simple in the model, i.e., the number of known physical concepts which describe a gas is considerably smaller than the number of variables which may influence the attitude of a consumer towards the purchase of a product in the market.

Therefore, the problem of constructing a model for an ideal gas is reduced to obtaining a functional relationship between the specific variables, pressure, temperature, and volume.

Consider a rectangular box of volume v containing n molecules. The gas exerts a pressure by the bombardment of its molecules on the walls of the box. Since the molecules move in all directions, and since there is no preferred direction, the pressure on any one face of the box is the same as the pressure on the other faces. By definition, the pressure P exerted by the molecules is equal to the rate of change of momentum per square centimeter of wall.

If \bar{x} is the mean velocity component in the x direction, the change of momentum resulting from the impact of a single molecule under consideration is $2m\bar{x}$, where m is the mass of the molecule. All molecules

within a distance \bar{x} should reach each square centimeter of the wall in unit time. Since there are n molecules in the volume v , it follows that $\frac{n\bar{x}}{v}$ molecules strike the wall in unit time. Therefore

$$P = \frac{2mn\bar{x}^2}{v}$$

where

$$\bar{x}^2 = \frac{kT}{2m}$$

$$P = \left(\frac{2mn}{v}\right) \left(\frac{kT}{2m}\right) = \frac{nkT}{v}$$

If N is the number of molecules in 1 mole, and V is the corresponding volume, it follows that

$$P = \frac{NkT}{V} = \frac{RT}{V}$$

$$R = Nk$$

Therefore

$$(1.2-1) \quad PV = RT$$

Equation 1.2-1 is the equation of state for an ideal gas and is frequently called the ideal gas law. Actually equation 1.2-1 is a natural science model, i.e., it is a symbolic statement expressing a unique relationship between specified variables. It can be compared with the Leontief model, $X = (I-A)^{-1}Y$, which is described in Chapter 2.

Assumptions of the Ideal Gas Model.-- The construction of the ideal gas model involves several assumptions which are analogous to the assumption of constant production coefficients and homogeneous output in the Leontief model. The basic assumptions of the ideal gas model are:

- 1) Intermolecular forces are negligible.
- 2) The volume of each molecule is a lot smaller than the volume of the space it inhabits.
- 3) Different forms of energy are separable.
- 4) The energy of the system is a quadratic function of its corresponding momentum.

Analogies Between the Natural Science Models and Marketing Models

The aim of the previous section was to present a description of a typical model in the natural sciences in a non-technical manner. The assumptions could have been stated in a more rigorous fashion by the use of mathematics. However, the entire discussion is presented merely as a foundation for the topic of this section which is an examination of the marketing interpretations of the assumptions of the ideal gas model.

Assumption 1. The existence of liquids is a real example of the fact that attractive intermolecular forces do exist between molecules even though they do not interact chemically. These forces are usually called Van der Waals forces. However, there are a few real gases in which the intermolecular forces are so small that the Van der Waals forces can be neglected. Therefore, although an ideal gas is nonexistent, the model can be used to explain the physical characteristics of real gases by relating them to deviations from the ideal gas.

An analogous assumption in the construction of marketing models is an absence of interaction between the component parts. In very few situations are the marketing activities of one firm independent of the actions of others. Therefore, a marketing model which

utilizes this assumption of non-interaction might be internally consistent, as is the ideal gas model, but it would have only a limited degree of applicability in the solution of marketing problems, whereas the ideal gas model can be used in the physical sciences.

Assumption 2. The assumption of the small size of each individual atom as compared to the space which it inhabits is analogous to the marketing assumption of pure competition, i.e., each firm in the model is so small that it cannot influence price. In the majority of the industrial applications of the ideal gas model the assumption of the relative smallness of the atom is not overly restrictive. Similarly, the assumption of pure competition is not a serious constraint on the general applicability of Leontief models.

Assumption 3. This assumption states that the total energy of a system may be separated into its component parts, translational, vibrational, and rotational. The actual separation of the total energy of any system into these unique categories is an empirical impossibility. Similarly, the division of all industries in the economy into sectors which produce a homogeneous product, as done in the Leontief model, is also an empirical impossibility.

Assumption 4. This assumption states that the energy of a system is a quadratic function of its corresponding momentum. The importance of this assumption should not be underestimated. Glasstone, an authority in physical chemistry, states that "the equipartition principle (which leads to the calculation of \bar{x}) depends on the particular form of the energy being an exact quadratic function of the

corresponding coordinate or momentum equation. If this is not the case, the principle must inevitably fail, even at high temperatures."⁴

A similar relationship in Leontief models is the assumption of a linear relationship between input and output. Fortunately in the case of the Leontief model, the model will not fail in the absence of this linear condition. However, due to the lack of data on the nature of production coefficients by industry type, the assumption of constant production coefficients is justified.

Basic Methodological Differences Between the Construction of Natural Science and Marketing Models

The previous section contains an analysis of the similarities between the ideal gas model, one of the more popular models in the natural sciences, and the Leontief model, probably the most widely used econometric model. However, even with this similarity in the nature of the assumptions used in both models, there still exist some large methodological differences:

1. The application of linear systems, e.g., network analysis, mass transfer systems, and linear equations of state, has more validity in physical science models than they have in marketing models.
2. The number of variables in the physical science models is smaller than the number used in marketing models, i.e., the specification problem is more complex in the construction of social science models.
3. There is a higher amount of noise (uncertainty) in marketing models than there is in the natural science models.

⁴S. Glasstone, Theoretical Chemistry (New York: D. Van Nostrand, 1961), p. 302.

4. The market analyst is often concerned with transient analysis. While engineering models also deal with this subject, generally the classical models of the natural sciences do not.

5. The most rewarding models that may be used by the market analyst are closed-loop systems, i.e., they allow for information-feedback in the system. Classical models in the natural sciences are usually open loop systems.

6. Many of the natural science models are micro models; for example, the ideal gas model was concerned with the bombardment of the sides of a cell wall by individual molecules. However, many marketing models are concerned with aggregate levels of economic activity.

7. The physicist usually deals with large numbers of elements in his models so that the introduction of one additional element may be neglected. Consider the Maxwell-Boltzman distribution law which gives the most probable distribution of molecules among the various possible individual energy values, at statistical equilibrium, for a system of constant total energy. In the derivation of this law the Sterling approximation is used. Since n is very large, an assumption is made that $n + 1/2$ is nearly equal to n , i.e., $(n + 1/2) \ln n = n \ln n$.

Unfortunately, the market analyst is not always in such an ideal position. Usually, he is constructing models in an oligopolistic market in which the number of firms are small and hence have a direct effect on the marketing activities of the other firms.

8. Some of the earlier models in classical mechanics assume that the position and the value of a moving particle can be uniquely determined at a given time. Similar conditions never exist in the development of a marketing model. The dynamics of particle movement in classical mechanic models can be compared with the changing preferences of consumers. However, to determine the unique position of a consumer on his preference scale is impossible without making a probability statement, i.e., the market analyst can designate the "neighborhood" in which the consumers' particular preference level exists, but he cannot give it a unique position.

The theoretical physicist argues that this criticism

of the lack of probability statements in classical mechanics is unwarranted with the development of wave mechanics, commonly referred to as quantum mechanics. To a degree the argument of the physicists is correct.

Frequently, to determine the position and the velocity of a moving particle, a beam of light is directed upon it. The earlier models of classical mechanics assume that the particle is of macroscopic size so that the momentum of the particle is not changed. However, with the introduction of electrons and atomic nuclei into theoretical physics, the particle is no longer macroscopic but microscopic. With the presence of microscopic particles in the system, a beam of light would alter its momentum.

With the uncertainty introduced into the analysis, due to the deflection of the particle by the light beam, the physicists turned to probability statements regarding the position and the velocity of a moving particle. The results of their investigations have been generalized by Heisenberg in the form of the Uncertainty Principle which states that if p and q represent two conjugate variables, such as momentum and position of any particle, the product of the uncertainties Δp and Δq in the determination of their respective values is approximately equal to the Planck constant, i.e.,

$$(\Delta p)(\Delta q) \approx h.$$

Even with the introduction of stochastic elements into the classical model, it still cannot be applied directly to marketing problems. The Uncertainty Principle will offer estimates of the relative uncertainty in conjugate observations given the Planck Constant. Unfortunately, there are no such parameters which describe marketing phenomena. At best there are certain structural parameters which describe different types of marketing activities; however, they are not invariant with time and would not have the same position in marketing models as does the Planck constant in quantum mechanic models.

Mathematical Models in Marketing

The author knows of no textbook which describes the basic principles underlying the construction of marketing models. Therefore the purpose of the remainder of this thesis is twofold; namely, to offer

a rigorous statement of the basic principles which a marketing model should meet and then offer an illustrative example of the method in which these principles are used in the construction of a mathematical marketing model.

In the past there has been considerable skepticism regarding the use of models in marketing. Many members of the "old school" believe that mathematical models similar to those used in the natural sciences have no role in marketing theory since marketing phenomena are a result of humans who are not amenable to mathematical law.

Another argument against the use of models in marketing is that intuition and past experience are the most important determinants of marketing decisions. Therefore, since the intuition of an individual is not likely to be transformed into a mathematical equation, the use of marketing models in actual business situations is of little value.

To the mathematically trained, the second argument has little foundation. If the intuition of an individual is reliable enough to generate a series of successful judgements, i.e., if there is a unique correspondence between a given event and a given outcome, then this relationship can be expressed by a mathematical relation. The only problem is that the relationship between the event and the action may be so complicated that the model used to represent it might have to be oversimplified, thereby, generating crude estimates of the actual situations.

The first criticism regarding the incompatibility of human behavior and mathematical models is out of context. First, no one

mathematical model is intended to describe all marketing phenomena. What is needed is a plurality of models, i.e., a series of models each of which describes one particular facet of human behavior in the market. "Mathematics," said the American Physicist Gibbs, "is a language." "If this is true any meaningful proposition can be expressed in a suitable mathematical form, and any generalizations about social behavior can be formulated mathematically."⁵

If one accepts the idea that mathematics is a language, then the usefulness of mathematics in marketing should not be questioned. It is the belief of the author that if marketing is to generate any true principles which are not tautologies and which are to have direct empirical use, then the use of mathematical models is inevitable.

The Classification of Marketing Models

Models may be classified in a variety of ways depending on the discipline in which they are used. The following table represents a possible classification of marketing models. It was constructed from a synthesis of the major models used in economics with some of the more important conceptual models used in the physical sciences.

Abstract or Physical.-- Physical models are replicas of some actual object or group of objects. An example of a physical model in marketing is a scale model of stockrooms available for inventory. The manipulation of prototypes representing certain types of merchandise aids in an effective use of existing facilities.

⁵D. Lerner and H. D. Lasswell (ed.), The Policy Sciences (Stanford, Calif.: Stanford University Press, 1951), p. 129.

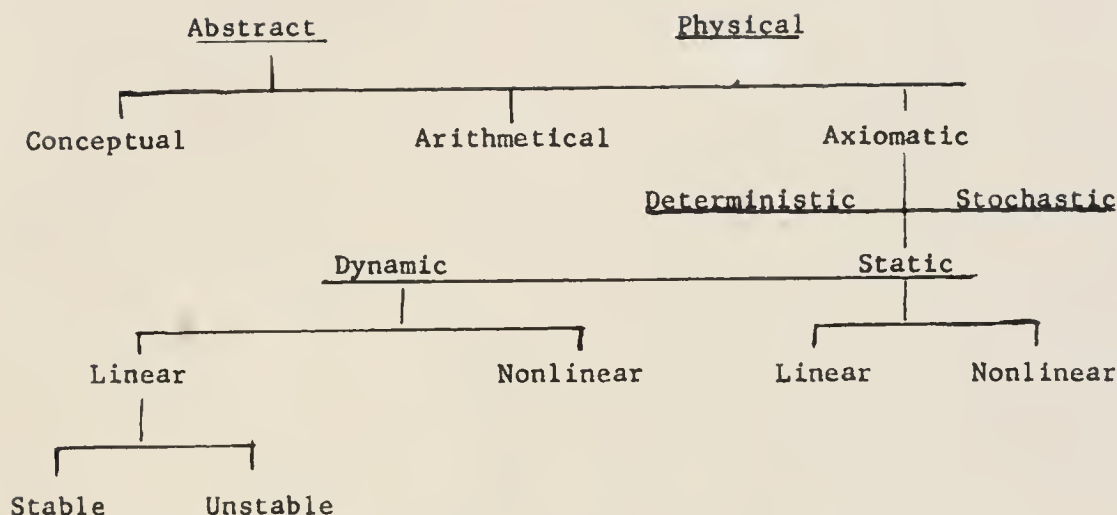


Figure 1.2-1. A Schematic Classification of Marketing Models

An abstract model is one which uses symbols or concepts to describe some segment of the marketing process. Reilley's Law of Retail Gravitation, which states that two cities attract retail trade from any intermediate city in the vicinity of the breaking point approximately in direct proportion to the populations of the two cities and in inverse proportion to the square of the distances from these two cities to the intermediate town, is an example of an abstract model.⁶

Conceptual, Arithmetical, or Axiomatic.-- Conceptual models are those stated in the terms of marketing principles. For example, "Advertising by itself serves not so much to increase demand for a product as

⁶E. J. Kelley, and W. Lazer, Managerial Marketing (Homewood, Ill.: R. D. Irwin, 1958), p. 422.

to speed up the expansion of a demand that would come from favoring conditions, or to retard adverse demand trends due to unfavorable conditions," is an example of a conceptual model.⁷

Arithmetical models are constructed by counting results. For example, many retail establishments use an automatic re-order plan. Such a plan involves the automatic reordering of certain types of merchandise when the inventory falls below a previously designated level.

Axiomatic models are mathematical statements which are deduced from a set of axioms or unproved statements. They are the form of "if X exists then Y follows." An example of an axiomatic model used in market analysis is the Leontief model which is described in detail in the subsequent chapters. It is the belief of the author that axiomatic models will lead to the greatest contributions to marketing analysis. An acceptance of Figure 1.2-1 substantiates this conclusion. The table shows that the more important mathematical models are an extension of the axiomatic model.

Deterministic or Stochastic.-- Deterministic models may be traced back to the early models of Newtonian mechanics. In essence these models show an absence of probability statements. In marketing, deterministic models are those taking place under absolute certainty. Deterministic models are compatible with those marketing models which assume perfect competition in the market since under this market structure, each entrepreneur has complete knowledge of his production costs and the demand for his product at a given instant of time.

⁷R. Bartels, The Development of Marketing Thought (Homewood, Ill.: R. D. Irwin, 1962), p. 60.

With the development of quantum mechanics, probability theory played a dominant role in the development of models in the natural sciences. Similarly, with the consideration of oligopolistic market structures, the market analyst was forced to consider decision making under conditions of uncertainty, i.e., the entrepreneur no longer had a complete knowledge of the market conditions at a given instant of time. For example, the demand for consumer goods may respond to expected (uncertain) prices instead of known prices. In order to treat the various uncertainties in the market, the market analyst is forced to develop models with random variables, i.e., stochastic models.

Static or Dynamic.-- Static models are concerned with variables which are not expressed as a function of time. This does not mean that the variables do not change with time, it merely means that the observations are taken at a given instant of time. Static models show their greatest validity in short-run models in which the importance of the time element may be minimized.

Dynamic models are concerned with an explicit consideration of the effect of time on the component variables. Dynamic models are of particular interest in studying the changes in industrial capacity since these are usually long-run phenomena in which time plays an influential role.

Linear or Nonlinear.-- Linear models possess two unique characteristics; namely, additivity and homogeneity. If a variable x_1 produces an effect a_1 when it is used alone, and variable x_2 produces an

effect a_2 when it is used alone, and if both variables x_1 and x_2 used together produce an effect equal to $a_1 + a_2$, then the variables are said to be additive. Similarly, if the variable x_1 produces an effect a_1 and if k_1x_1 produces an effect k_1a_1 , then x_1 is homogeneous.⁸

Absence of either additivity or homogeneity denotes nonlinearity. Usually nonlinear models offer a more accurate description of reality. Unfortunately, the mathematical analysis involved in nonlinear systems is very complex since general solutions to nonlinear systems are nearly impossible to obtain.

Stable or Unstable.-- The problem of stability is an auxiliary problem of dynamic models. A system or model is stable if after being subjected to some divergence around its equilibrium position, it returns to its initial or static condition. Similarly, a system is unstable if after a dynamic movement of its components it does not return to its equilibrium position.

The stability in marketing models can be related to supply and demand schedules for consumer goods. Equilibrium between these quantities occurs when the supply price equals the demand price. If a change in price is accompanied by a corresponding change in output, then the given equilibrium position is stable.

The Components of Axiomatic Marketing Models

Every marketing model regardless of the degree of its complexity has several common components, namely, undefined terms, definitions,

⁸G. Hadley, Linear Algebra (Reading, Mass.: Addison-Wesley, 1961), p. 2.

axioms, postulates, rules of inference, theorems, and lemmas. This section describes each of these components. The methodology used in the development of marketing models is deductive; a deductive system is one which begins with a group of assumptions from which theorems are eventually deduced.

Undefined Terms.-- Basic to any marketing model are the introduction and use of some undefined terms. The ambitious but mathematically naive market analyst may attempt to define all the terms in the model. Such an attempt will give rise to two different situations.

First, the definition of one term will lead to the introduction of additional terms which must then be defined. This process may continue until there is an infinite number of terms in the model. Second, an attempt may be made to define each term by relating it to the existing variables in the system. The result of this approach will lead to a circularity of definitions which offer no information. "In general no attempt is made to analyze undefined terms; they are accepted as given."⁹

Definitions.-- As the model increases in complexity new concepts are introduced. In order to offer a rigorous statement of these concepts, they are defined in terms of the undefinables. For example, Euclid's definition of a line is that which has no breadth. In this definition of a line, breadth is taken as an undefinable.

⁹J. R. Feibleman, "Mathematics and Its Applications in the Sciences," Philosophy of Science, XXIII (1956), 204.

Axioms.-- Both the undefined and the defined terms are combined into statements known as axioms. "Axioms are unproved statements which are never submitted to any test of truth. The axioms taken together are known as the axiom-set. It is the purpose of the axiom-set to yield a great many theorems."¹⁰ In essence the axioms of a model represent the underlying assumptions upon which it is constructed.

Postulates.-- Both postulates and axioms comprise all the assumed statements of a model. The test determining whether a certain assumption is an axiom or a postulate will be defined as follows: "If the statement contains no undefinables or defined terms of the science itself, but only the terms of presupposed sciences, then it is an axiom; but if it contains a term which is an undefinable of the system or is defined by the undefinables of the system, then it is a postulate."¹¹

Rules of Inference.-- After the set of undefined terms and axiom-sets are formulated, they must be manipulated in such a way as to lead to the derivation of theorems and lemmas. There are two generally accepted rules of inference, namely those of substitution and detachment. The principle of substitution states that equals may be substituted for equals. Detachment implies that whatever follows from a true proposition is true.

Theorems and Lemmas.-- Theorems are statements which are deduced from the axioms, postulates, definitions and undefined terms, of the system. In mathematical systems, these statements are usually written

¹⁰Ibid., p. 205.

¹¹C. W. Churchman, Elements of Logic and Formal Science (New York: J. B. Lippincott Co., 1940), p.10.

in symbolic form. In marketing, the mathematical statements of these deduced theorems are called models. A lemma is an auxiliary or supporting theorem which is proved in order that the principal theorem may be proved.

Necessary Steps in the Construction of Marketing Models

Figure 1.2-2 is a schematic representation of the steps involved in the construction of marketing models. The primary steps are those of specification, estimation, verification, and prediction. These steps are perfectly general and generate the basic format for the construction of many types of models. These steps are used in both the construction of natural science models and econometric models.¹² The methodology illustrated in Figure 1.2-2 is not exhaustive; however, it does represent the important steps involved in the construction of marketing models.

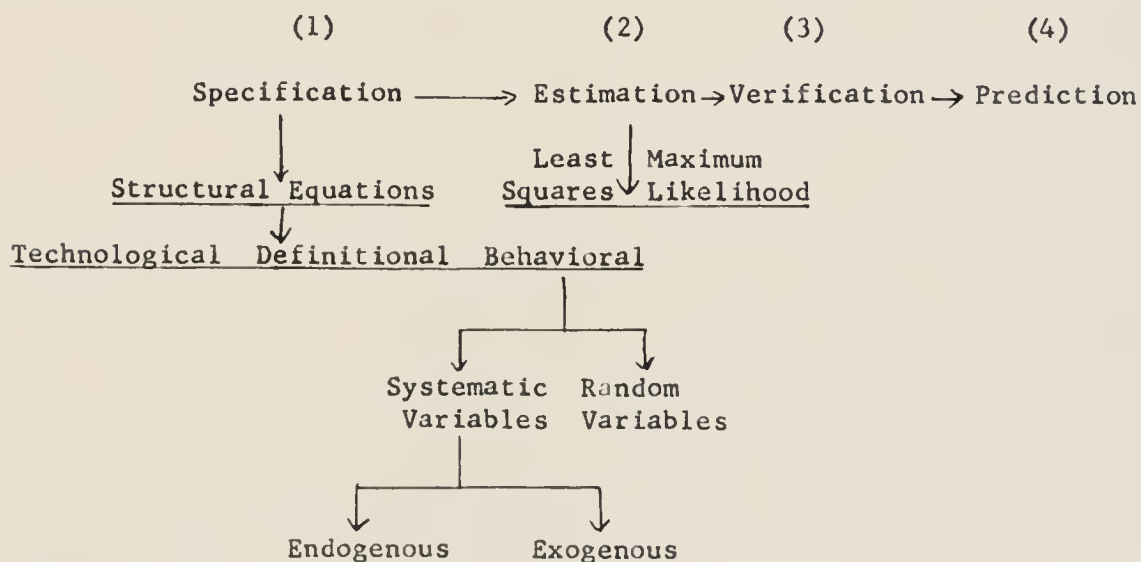


Figure 1.2-2. Model Building

¹²Valavanis,

Specification

Specification is the most basic step in the model building process. In economics, the specification problem is usually solved by the mathematical economist whereas the econometrician is usually interested in the last three steps: estimation, verification, and prediction. In the construction of a marketing model, specification is the process by which a marketing theory is developed and then expressed in symbolic form. Accuracy in specification is of utmost importance since it involves an explicit consideration of those variables to be included in the model. In this step of the model building process, the market analyst must make a choice as to which variables, out of the countless number that describe marketing activity, he believes to be the most important for the solution of the given problem.

The pinpointing of the variables to be considered in the model is only the first step toward the completion of the specification problem. The next step involves the formation of a quantitative relationship between the selected variables. The mathematical relationships between the pertinent variables are called structural equations because they describe the basic framework of the model.

Technological, Definitional, and Behavioral Equations.-- Each structural equation may be classified into one of three classes; namely, technological, definitional, or behavioral. Technological equations are classified according to the nature of their constraints; if the constraints on the equation are determined by the technological

conditions in the market, then the structural equation is a technological equation. Technological equations are transformation functions, i.e., given a value for the independent variable, the equation makes a mechanical transformation on it with the result that the value of the dependent variable is uniquely determined. The production function discussed in neo-classical economics is an example of a technological equation.

Definitional equations owe their existence to the identities used in the model. They represent truisms which are invariant with time. Unlike the technological or the behavioral equations, the definitional equations are not influenced by the conditions of the market. An example of a definitional equation used in marketing is that the total flow of consumer goods is equal to the sum of the individual flows of specialty, consumer and convenience goods.

Behavioral equations describe the actions of the fundamental units, households, and business firms, in the market. Examples of behavioral equations are the demand curves for the consumer and the profit maximization equation for the firm. Generally, the accuracy of the behavioral equations exerts the greatest influence on the accuracy of the entire model since the error in the technological or definitional equations is not likely to be as large as the error in the behavioral equations.

Systematic and Random Variables.-- A set of variables is systematic if the variables are defined in terms of each other. For example, if Y (the dependent variable) is always equal to a times X , the value

of Y is related to the value of X in a unique manner. A random variable is one which has a probability distribution and is the independent variable in the system.

Usually, the proper use of random variables increases the realism in marketing models. For example, assume that the demand function for some consumer goods is of the following form: $D = f(P)$. Although demand is a function of price, it is also a function of other variables such as income, taste, and prices of substitutable quantities. Therefore, the equation may be written as:

$D = f(P) + e$, where e designates the variation due to these "other" forces.

Endogenous and Exogenous Variables.-- All systematic variables must be either endogenous or exogenous. Endogenous variables are determined by the model whereas exogenous variables are determined by forces outside the model. Therefore, the values of the exogenous variables are assumed to be given. Exogenous variables are independent variables whereas endogenous variables are dependent.

Closed models are self-contained, i.e., all the variables are endogenous. Therefore values for all the variables are determined within the model. Open models employ both exogenous and endogenous variables. The exogenous variables relate behavioral patterns which are external to the system to the endogenous variables.

Estimation

In the first step of the model building process, specification,

pertinent variables (D and P in the previous example) are selected and then related in a unique manner. Estimation is the second step in the model building process. Every model contains certain structural parameters whose values must be estimated. A demand equation maybe of the form

$$D = aP + b$$

contains the structural parameters a and b and the specified variables D and P. It is beyond the scope of this thesis to go into an elaborate description of the statistical techniques of estimation since several excellent books have been written on this subject.¹³ Two of the most fruitful approaches are the use of least squares and maximum likelihood methods.

Verification

Verification, the third step in the model building process, involves a study of the accuracy of the preceding steps. The results generated by the model are compared with observed values and their difference is recorded. In order to determine if this difference is significant certain statistical tests must be employed and a criterion for acceptability must be designated.

Verification is an important step since it gives the market analyst some idea of the accuracy of the results generated by the model. It gives him additional information so that he can express the results of the model in terms of probability statements.

¹³Valavanis; T. C. Koopmans (ed.). Statistical Inference in Dynamic Economic Models (New York: John Wiley, 1950).

Prediction

Prediction, the fourth step in the model building process, is primarily an organizational step. It involves a rearranging of the model so that it may be suited for empirical studies. A typical action in this step involves a statement of the model so that it may be programmed on a large scale electronic computer.

Outline for Subsequent Chapters

The following chapters contain an application of the Leontief interindustry model to marketing. The general outline follows the previously discussed steps in the model building process. The next chapter contains the basic theory underlying the construction of the model. This is the step of specification. Following this step the next chapter discusses the statistical estimation of pertinent parameters. The final chapters contain studies concerning the validity of the model which is the important part of verification. The model is then used to predict industry outputs in Duval County, Florida.

CHAPTER 2

THE THEORY OF INPUT-OUTPUT MODELS

2.1 Historical Note

Economic historians usually state that Quesnay developed the first and most basic input-output model when he published the Tableau Economique in 1758. Schumpeter states that Cantillon was the originator of the table. Although Cantillon did not present his analysis in tabular form, his Tableau is essentially the same as Quesnay's.¹

Quesnay's use of the table was quite different from its present use. His purpose in formulating the table was to show that only the agricultural industry produced a net profit. Presently, the primary interest in the Tableau is due to its contribution to economic methodology; therefore, the conclusions drawn by Quesnay are not examined. Probably the greatest contribution of the Tableau to economic-analysis was that it presented for the first time, a general theory of economic equilibrium. It showed the relationship between money flows and the flows of goods and services, thus illustrating the interdependence between the various sectors of the economy. This concept of general economic equilibrium was to have far reaching effects on the writing of future economists.

¹J. A. Schumpeter, History of Economic Analysis (New York: Oxford University Press, 1954), p. 222.

After the publishing of the Tableau, there were no significant contributions to general equilibrium theory until 1874 at which time Walras published his monumental Elements d'Economie Politique Pure. The details of Walras's Model are discussed in the following section. In essence the model determines the prices and the consumer demand for economic goods, given their supply and demand schedules, production functions, and the utility functions of the consumers. Walras is often criticized for not presenting a model which could be substantiated by empirical analysis. The difficulty in using the model for the solution of actual economic problems is apparent if an attempt is made to obtain all the relevant information for the construction of the utility functions for each consumer and the supply and demand schedules for all of the commodities in the economy. However, Walras was an economic theorist and did not intend that the model be used for the solution of empirical problems. His objective was to show the structural relationships between pertinent economic variables and to use these general relationships for an illustration of the interdependency between the various sectors within an economy.

It was not until the publication of The Structure of the American Economy, 1919-1929 by Professor Leontief that the theoretical system of Walras was modified in order to provide solutions to some of the most difficult problems ever encountered by economists. The technical differences between the Leontief and Walrasian models are the topic of the next section. The input-output model of Leontief is one of three basic models used in interindustry economics. The other two are linear programming and process analysis.

"Input-output analysis or the quantitative analysis of inter-industry relations has in recent years absorbed more funds and more professional resources than any other single field of applied economics."² However, the model has not been accepted by many members of the profession. Ironically, the General Theory by Keynes and Leontief's paper Quantitative Input and Output Relations in the Economic System of the United States were both published in 1936. Both the works of Keynes and Leontief have made notable contributions to economic analysis. The spectacular success of the Keynesian model is now compared with the limited success of the Leontief model.

First, the statistical methods involved in verifying the Keynesian model are trivial compared to those of Leontief's Model. The simplicity in the mathematical statement of the Keynesian model permits the use of basic statistical tools in determining its validity. However, in the case of the Leontief model, empirical verification is quite difficult. The data required for the execution of the Leontief model are of enormous magnitudes resulting in large expenditures of human and financial resources. With the development of the electronic computer the statistical advantage enjoyed by the Keynesian model diminishes.

Another factor leading to the increased popularity of the Keynesian model over that of Leontief's is that the Keynesian model deals with the problem of unemployment, which was of primary interest

²National Bureau of Economic Research, Input-Output Analysis: An Appraisal, Conference on Research and Wealth (Princeton, N.J.: Princeton University Press, 1955), p. 3.

in the thirties. The Keynesian model presented a possible solution to a current problem whereas Leontief's model was concerned with the balance between industrial output and consumer demand.

The third factor contributing to the popularity of the Keynesian model over that of Leontief's is that Keynes, as did many of his readers, believed that his model was revolutionary since it appeared to be in direct contradiction to many of the prevailing neo-classical theories. Hence his model provided the anti-neo-classicists with a nucleus around which they could expound their theories. Leontief's model, which was formulated on the theories of the classical and neo-classical economists, was not of this nature. Leontief's model did not refute the Walrasian model but it did simplify the model so that it could be used as an empirical tool for economic analysis. Some economists doubted that the Leontief model could be used in an empirical analysis and therefore classified it as a model which was not as general as that of Walras in addition to having the same empirical disadvantages.

Although the two models were competing for popularity, the previous discussion should not imply that they were totally unrelated. Both models are involved with an efficient allocation of resources, hence their ends are identical. The differences between the two models are in their means to these ends. The Keynesian model is concerned with the effective allocation of resources by the use of monetary and fiscal policies. The Leontief model is also interested in such an

allocation and attempts to do so by co-ordinating industrial output with consumer demand. A proper use of the Keynesian model provides a solution to the cyclical unemployment problem. The Leontief model provides a means for maintaining this level of employment.

2.2 Leontief's Model: A Special Case of the Walrasian Model

Schumpeter refers to the Walrasian model as the "Magna Carta of Economics" and this is exactly what it is. In this model Walras gave a precise theoretical statement of the manner in which the prices and the quantities of goods and services demanded in the market are determined. His approach is to be distinguished from the partial equilibrium approach of Marshall.

"Economics, like every other science, started with the investigation of local relations between two or more economic quantities, such as the relation between the price of a commodity and the quantity that is available in the market; in other words, it starts with a partial analysis,"³ A general equilibrium theory explains the interdependence between all economic phenomena. Marshall admits that his theory is one of partial equilibrium; unfortunately Keynes's is not as explicit. The General Theory of Keynes does not offer an explanation of equilibrium throughout the entire economy; it concentrates on one sector of the economy and is primarily interested in the equilibrium of national aggregates. Marshall's approach is usually classified as a partial equilibrium analysis since it is concerned with the determination of

³Schumpeter, p. 242.

prices for the output of a firm which is too small to affect national aggregates such as national income and total employment.

It is interesting to examine Marshall's contribution to the theory of general equilibrium. Quite frequently Marshall is accused of not contributing any new tools to economic analysis. Supposedly, he merely synthesized the theories of the classical and marginal utility schools. An example of incorrectness of this statement is given in the Mathematical Appendix of the Principles. In this discussion he is using a system of simultaneous equations to show that the wage received by a carpenter tends to equal the marginal productivity of his output. He then states,

It would be possible to extend the scope of such systems of equations as we have been considering, and to increase their detail, until they embraced within themselves the whole of the demand side of the problem of distribution. But while a mathematical illustration of the mode of action of a definite set of courses may be complete in itself, and strictly accurate within its clearly defined limits, it is otherwise with any attempt to grasp the whole of a complex problem of real life, or even any considerable part of it in a series of equations.⁴

It can be concluded that Marshall was aware of the relation of his partial analysis to a general theory of equilibrium. However, he was doubtful of the results of such an approach; this is obvious when he referred to Walras only three times in his book, all of which were brief unimportant statements.

Walras's System of Equations

The system of equations used by Walras represents his model.

⁴Alfred Marshall, Principles of Economics, 8th ed. (New York: Macmillan Co., 1948), p. 850.

It is important to note that these equations, which constitute his general equilibrium model, are concerned with the general equilibrium of production and exchange. Basic to this model is the assumption that all firms are so small that they cannot influence prices; i.e., the model is developed under the conditions of pure competition. Walras's system of equations describes an economy in which each consumer maximizes his material welfare with the land, labor, and capital which are available to him. Walras's model does not have any assumptions regarding the distribution of wealth, it merely describes the equilibrium determination of prices.

The Walrasian model can be represented by a set of simultaneous equations. Let there be n factors of production (productive services) used in the production of m products. These n factors and m products will be subject to the following conditions:

Condition 1

The quantity of each productive service demanded is equal to the quantity supplied.

Condition 2

The quantity of each productive service supplied is a function of its price.

Condition 3

The price of each product is equal to its cost of production.

Condition 4

The quantity of each product demanded is a function of its price.

In Conditions 1 and 2 there are n equations since there are n factors of production. Conditions 3 and 4 both have m equations since there are m products. Therefore the total number of equations is $n + n + m + m = 2n + 2m$. However, Walras expresses the prices of all goods in terms of a unit account called a numeraire whose price is equated to unity. Since the price of this commodity is given by definition, there are only $2n + 2m - 1$ unknowns ($n + n + m + m - 1$). Hence the four conditions generate a system of simultaneous equations in which there are $2n + 2m$ equations in $2n + 2m - 1$ unknowns. This problem can be solved by recognizing that the m equations in Condition 4 are not independent. Therefore one of these m equations can be written for the numeraire. Since the price of this product is given, the equation for the numeraire in Condition 4 can be derived from the equations for it in Conditions 1 and 3. Therefore, the system reduces to $2m - 2n - 1$ equations in $2m - 2n - 1$ unknowns.

Although the number of equations is equal to the number of unknowns, the solution may not be unique, i.e., there may be no solution or there may be an infinite number. Mistakes of this nature are frequently found in the literature. Marshall is guilty of this mistake. He states, "However complex the problem may become, we can see that it is theoretically determinate, because the number of unknowns is always exactly equal to the number of equations we obtain."⁵ The necessary mathematical properties of a system of simultaneous equations, which insure a unique solution to the system are discussed later on in this thesis.

⁵Ibid., p. 856.

The eminent mathematician, Abraham Wald, modified the Walrasian model and stated the conditions under which the system would have a unique solution. Wald concluded that the quantities and sources of both the products and the factors of production can be determined given the form of the demand and supply functions for the market, the number of factors and goods in the system, the coefficients of production and the amounts of productive services held by the individuals at the beginning of the period.⁶ The complexity of the system increases as soon as non-linear equations enter the model. In this case, the equations may support the thesis of consistent equilibrium but there may be no real solutions. Even if the equilibrium position does exist, there is no guarantee of its stability.

The problem of stability is discussed in Appendix A but a few remarks are in order. Usually the problem of stability is studied by a partial analysis, i.e., all of the variables in the static model are fixed except one. By allowing one variable to change within a known range, its effects on the other variables in the system can be determined. This method of partial analysis in general equilibrium models is one of Leontief's great contributions.

Deficiencies in the Walrasian System

This section contains a discussion of the shortcomings of the Walrasian model. Walras was aware of these deficiencies, however, he had to make certain assumptions in order to construct the model. The

⁶O. Morgenstern, Economic Activity Analysis (New York: John Wiley, 1954) p. 13.

restrictions in the Walrasian model are of interest for several reasons. First, Walras's model is one of the first models of general equilibrium, therefore his model serves as a basis for many economic models. Unfortunately many of the deficiencies inherent in his model are passed on to other economic models. Second, with the recent advances in mathematical theory and electronic data collection, some of the restrictions may be eliminated and lead to a more general theory of equilibrium. Third, many of the restrictions may be solved by a partial equilibrium analysis; traditionally, economics has used this approach in lieu of a general equilibrium analysis. Therefore, an examination of the restrictions and limitations of the Walrasian model may lead to a closing of the widening gap between the mathematical economist and his normative colleagues.

Linear Homogeneous Production Functions.-- The Walrasian system is determinate, if the production functions are linear homogeneous entities. The assumption of linear homogeneous production functions is an excellent example of the deficiencies in the Walrasian model which are passed on to other models, the most striking example being the Leontief model. Consider the following production function

$$x = f(x_1, x_2, \dots, x_n)$$

where x is equal to the output produced from the input (x_1, x_2, \dots, x_n) .

"A function f of n variables x_1, x_2, \dots, x_n is called homogeneous of degree m , if upon replacement of each of the variables by an arbitrary parameter k times the variable, the function is multiplied by k^m ."⁷

⁷I. S. Sokolnikoff, Advanced Calculus (New York: McGraw-Hill, 1939), p. 75.

Therefore, the above production function is homogeneous of degree m if $f(kx_1, kx_2, \dots, kx_n) = k^m f(x_1, x_2, \dots, x_n)$.

This implies that

$$x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} \dots x_n \frac{\partial f}{\partial x_n} = m k^{(m-1)} f(x_1, x_2, \dots, x_n)$$

If the production function is homogeneous

$$x = k^m f(x_1, x_2, \dots, x_n)$$

hence

$$\frac{\partial x}{\partial k} \text{ is a constant.}$$

The economic interpretation of this result is that a linear homogeneous production function leads to a constant return to scale. The above analysis assumes that the partial derivatives are continuous, i.e., the inputs and outputs are variable so as to lead to a continuous production function. This implies that there is a continuous substitution of the factors of production. The Walrasian model does not stop with the assumption of constant returns to scale, it also assumes that the production coefficients are constant. Generally with the assumption of constant coefficients of production, there is no substitution between inputs in the production function. Consider the general function

$$x_{ij} = a_{ij} X_j$$

The term a_{ij} is the technical coefficient which is assumed to be fixed. The greatest disadvantage in assuming constant activity coefficients is that it prohibits substitutability among inputs and changes in productivity. The validity of this assumption is discussed when a similar assumption made by Leontief is subjected to an empirical investigation.

In both the Leontief and the Walrasian systems the firm has already made its choice regarding the nature of the production process to be used. Therefore, the assumption of fixed production coefficients is not as stringent as it appears to be. The development of linear programming has eased the rigidity of the constant productivity assumption. The application of linear programming techniques to all the possible processes available for the production of the output, leads to an optimum combination of the inputs, given some objective function to be maximized or minimized.

A linear homogeneous production function coupled with the assumption of constant activity coefficients will guarantee a unique solution to the Walrasian system; however, this is not the only condition for which the system is determinate. Rogin states that

with complete variability in the proportion of inputs (premised on complete divisibility and mobility of economic resources) Walras's mechanism of competition still assures a determinate solution for price and output in the markets for all commodities premised upon the determinate equilibrium output of each firm. The essential condition is that in the context of each firm the dosing of each factor service be accompanied by decreased incremental returns.⁸

Incremental returns should be distinguished from proportionate returns. The former refer to the rate of variation of output associated with the variation of input of a factor, the latter to the ratio of the proportional (percentage or average) change of output to the proportional (percentage or average) change of input.

⁸Leo Rogin, The Meaning and Validity of Economic Theory (New York: Harper Brothers, 1956), p. 439.

A Static Equilibrium Model.-- The model of Walras is a static equilibrium model since none of the equations is a function of time. This is a serious restraint since many sectors of the model are dependent on the time variable. Consider the investment sector. Future investments are a function of future expectations concerning the general price level and the interest rate. These future expectations are a function of time which is not present in the Walrasian model. Therefore, the elimination of the time element from the Walrasian model seriously restricts its use as an empirical tool of analysis.

Previously, the conditions were stated upon which the Walrasian model was constructed. Two of the conditions were an explicit statement of the assumption that supply and demand were a function of only price; thereby implying that these schedules were determined only by current prices and were invariant with future prices.

The Existence of a Unique Solution for the Walrasian System.-- Walras assumed that since his system had a number of independent and consistent equations equal to the number of unknowns the solution would be unique. However, the solution may be unique but there is no guarantee that the solution will be non-negative which is essential for a proper economic interpretation of the results. Wald modified the Walrasian system by ignoring the marginal utility functions and assuming that the factors of production were given. He then proved that if the following conditions are fulfilled the Walrasian system has a unique non-negative solution set.⁹

⁹R. E. Kuenne, "Walras, Leontief, and the Interdependence of Economic Activities," The Quarterly Journal of Economics, LXVIII (1954), 345.

1. The production coefficients are non-negative.
2. The supplies of the factors of production are non-negative.
3. At a minimum, one factor of production is used in the production of each commodity.
4. The rank of the activity matrix is equal to the number of productivity factors.
5. The demand functions are continuous and positive for any finite price.

Stability of the Walrasian Model.-- Another criticism of the Walrasian model is that it fails to relate the changes in the supply and demand functions to the original state of equilibrium. Walras was confused in his attempt to relate stability to the existence of a unique solution set. A static approach to both problems is overly restrictive, i.e., a unique solution to the system is not a sufficient criterion for stability. The problem of stability can be properly analyzed by the consideration of dynamic extensions of the model.

Walras did consider the effects of disturbances such as price changes in his system, but he failed to state explicitly the path of the transition. He did not describe the relation of the new equilibrium position to the original one. There is no guarantee that a divergence from the original equilibrium position in the Walrasian model will move back to the original static position.

The problem of divergences around a given equilibrium position stresses the need for a dynamic interpretation of the Walrasian model. A stable equilibrium position when subjected to disturbances is one in

which the end results of the divergences lead back to the static equilibrium position. However, these divergences are dynamic and must be subjected to one analysis; not by the static approach used by Walras in which he attempted to relate the existence of a unique solution to stability.

A Purely Competitive System.-- Basic to the construction of the Walrasian model is the idea of pure competition. The previous analysis contains a discussion of the determinacy of the system given this basic assumption. However, with the introduction of oligopolistic elements into the model, the solution is no longer determinate.

In the oligopolistic version of the Walrasian model the assumptions of fixed production coefficients and constant returns to scale, along with the other conditions previously described, no longer lead to a determinate solution for several reasons. First, there is an increase in the number of variables in the system due to the variety of products in an oligopolistic market. Therefore, depending on the nature of the model, many additional variables will be introduced. Second the structural relationships in the economy become complex due to the coalitions in the oligopolistic market. Many of the mathematical relationships in the model would have to be expressed as non-linear polynomials.

A partial answer to the oligopoly problem is to use game theory for the construction of the model. Conceptually game theory could deal with the problems introduced by labor unions, consumer cooperatives,

and other coalitions in the market. However, game theory as it exists today, is in a primitive stage and not directly applicable to the Walrasian model since the number of participants in the game are limited to a small number. However, it is simpler to construct a new model from the theory of games than it would be to modify the existing Walrasian model, so that it would be applicable to an oligopolistic market.

The Significance of Walras's Contribution

The previous discussion of the shortcomings in the Walrasian system should not imply that the model is not a valuable tool of economic analysis; it merely describes those assumptions in the model which limit its generality. One of the greatest contributions of the model is that it synthesized many of the partial equilibrium theories into one unified body of thought. For example, the theory of marginal productivity may be obtained from the general model of Walras by eliminating the assumption of constant production coefficients.

Partial equilibrium theorists are aware of the limitations of their analysis. Walras provided a model which would overcome the major objections of partial equilibrium analysis since his model considers the interdependence of economic phenomena. Although the various activities in the economy are related by naive mathematical functions, the general relationships do exist.

The model also specifies some of the more influential variables on the level of economic activity. The scope of the model can

be expanded to cover a larger number of sectors in the economy; however, extensions of this type would involve the addition of many variables to the model. Since the equilibrium model of Walras is general, these additional sectors may be added without any conceptual changes in the construction of the model.

The model of Leontief which is discussed in the next section, is derived from the Walrasian model. Essentially, Leontief converted the theoretical model of Walras into an empirical tool for economic analysis. A practical application of the Walrasian model is of extreme importance since it is useful for prediction, the goal of any science. In order to construct his model, Leontief had to make some general modifications of the Walrasian model which included:¹⁰

Aggregation.-- Walras did not consider the problem of aggregation, the details of which are discussed in a subsequent chapter. Without aggregating the input data to the model, each product and each factor of production within the model would need an equation. For example, if there were 1,000 products and 100 factors, which is very small for any realistic economy, there would be 2,199 equations and unknowns in the system. Even with high speed electronic computers, the solution to this system of equations would be tedious, if not impossible.

Marginal Utility Functions.-- Walras paid little attention to the accessibility of information on marginal utility functions which

¹⁰ O. Morgenstern, p. 22.

are difficult to measure in practice. Market demand and supply functions may be directly related to utility functions by assuming each consumer maximizes his utility.

Free Competition.-- Since the assumption of free competition is made in the Walrasian model, this divergence from reality should be corrected when the model is placed into use. One method is to use empirical demand, supply, and production functions. The advantage in this approach is that these functions are descriptions of reality; they can take into account the existence of partial and complete monopolies.

The Derivation of Leontief's Model from the Walrasian System

The object of this section is to show that a proper modification of the Walrasian model will lead to the Leontief model. Leontief's model is the first modification of Walras's general equilibrium model which can be subjected to empirical verification. However, this modification is achieved by a loss of generality. Walras's model is a closed static model since it does not contain an exogenous sector. An exogenous sector is one in which the flows are obtained from an independent analysis before they are used in the model. Leontief's model is open since it has an exogenous sector. One of the primary differences between the closed model, which has only an endogenous sector, and the open model, is that all of the production functions are not considered to be constant in the open model. Presently, research is being performed with the objective of closing the Leontief model in order to approach Walras' general theory of equilibrium.

In an early part of this section four conditions were stated as the basic foundations of the Walrasian system. Although not explicitly stated, but inherent in the analysis, was the assumption that each individual acted in such a way as to maximize his position (for example, profits, or utility). Walras also assumed that the institutional setting, quantity and quality of productive resources, consumer tastes, and other pertinent data were given.

Leontief made the following modifications to the Walrasian system:

1. Each industry produces only one product.
2. There is no explicit use of utility functions.
3. The quantities of factor services supplied are not expressed as a mathematical function of price.
4. The quantities of products demanded are not expressed as a mathematical function of price. They are usually placed in the final demand category and are taken as given data.
5. The technical coefficients of each industry are fixed.
6. The interest rate is known.
7. Labor is the only primary (non-produced) input.
8. The number of inputs is equal to the number of outputs.
9. For the period under consideration, the total output of any good is equal to the consumption of that period.
10. The input of capital is a fixed proportion of other inputs. Therefore, inventories may be counted as one of the other industries in the model or it may be included as an element of the final demand figure.

By applying these assumptions to the system of equations representing

the Walrasian system, Leontief derived the model which is described in the next section.

2.3 The Basic Model

Introduction

As an economic unit increases its productive capacity there is a tendency for the division of labor to show a comparable increase. An increase in the division of labor for any society produces an interdependence between the producing and consuming sectors. Interdependence is the system of relationships which describes the complex activities in an economy.

An infinite number of statements can be written to describe a particular segment of the economy. The combination of all statements leads to a general model of social behavior. The Leontief model does not pretend to have this universality, since it concentrates on the economic sector of society. The relationships in the model are not explicitly related to the ideals, motivations, or the noneconomic interest of the individuals within a society.

The primary objective of the model is to relate by quantitative relationships, the demand for finished products with the available agents of production, land, labor, and capital. In a complex industrial society, there is no established connection between the demand for products and the aggregate amount of resources required for the fulfillment of this demand. Usually the production of any product a_1 requires raw materials in the form of other products b_1, b_2, \dots, b_n .

However, the production of each of the original input products, b_1 , b_2 , . . . b_n requires input products of the form c_1 , c_2 . . . c_n . This process may continue ad infinitum. There are several advantages in using a mathematical model for expressing the complex chain reaction between inputs and outputs. The model is quite useful in forecasting output, employment, and income levels for a given level of demand. Information of this type indicates which sectors of the economy have an excess or deficiency in manpower and financial resources. For example, in the early thirties there was a controversy between the Works Project Administration and the Public Works Administration concerning the effect of different types of expenditures on employment levels. The Works Project Administration wished to spend the majority of its funds on wages and only a limited amount on materials. Conversely, the Public Works Administration was in favor of large expenditures for construction materials and limited expenditures on direct wages. To resolve the problem the Construction Division of the Bureau of Labor Statistics was asked to compare the effect of direct work relief expenditures and expenditures for public construction projects on employment levels. The use of basic interindustry techniques solved the problem.

In the last 20 years, considerable research has been undertaken to extend the use of Leontief's model. The first major extension was the application of linear programming techniques to input-output analysis. The use of this technique permits a consideration of the substitutability of inputs in the model in order to maximize some economic

objective function. Given a consumer demand schedule, there are a variety of input schedules which will meet the given demand requirements. The linear programming approach will generate a solution which determines the optimum input schedule given specific criteria to meet, such as maximum employment.

The second and most recent innovation to input-output analysis is the method of process analysis, which is concerned with a detailed study of the technical processes of production and relates such activities to output, employment, and income levels. Process analysis is not limited to the use of linear models. However, the majority of work to date involves such an assumption.

Transactions Table

Basic to any input-output model is a transactions table. The table merely represents a double entry bookkeeping system applied to industry purchases and outputs. The transactions table is often called a transaction matrix since it leads to the development of equations which are usually expressed in matrix form. Table 2.3-1 is a schematic arrangement of a transaction table. For purposes of discussion, the table is divided into four arbitrary sections called quadrants which have no effect on the analysis.

Quadrant II.-- The total value of the output of industry 1 is represented by X_1 . The corresponding x_{12} represents the output of industry 1 that goes to industry 2; and x_{13} represents the output of industry 1 that goes into industry 3. The general term, x_{ij} , is the

output of industry i going to industry j . The same analysis applies to the industries in the final demand sector. Therefore an examination of all the x_{ij} 's describes the nature in which the output of each industry is distributed. This analysis may be applied to each row in the transactions table.

The preceding analysis is based on a consideration of the row entries in the table. However, a similar analysis may be applied to the column entries. In column 3, X_3 represents the total purchases of industry 3. In the analysis of row entries, it was concluded that x_{13} represented the value of goods produced by industry 1 and purchased by industry 3. In the analysis of column entries it represents the purchases of industry 3 from industry 1. Likewise x_{23} , which is the next entry in column 3, represents the purchases of industry 3 from industry 2. An analysis of column entries shows the distribution of an industry's purchases whereas the corresponding row entries show the distribution of the industry's output.

An examination of the table shows that the total output of an industry is equal to its total purchases. This occurs if the inter-industry flows are measured in money terms, not physical terms. If such a convention were not adopted, column totals would have no meaning since the output of one industry may be in tons and the other in thousands of units. Therefore, since every sale is a purchase, row and column totals are equal. In a later discussion, the above condition is found to generate a matrix in which the columns are stochastic.

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FIGURE 2.3-1 A SCHEMATIC ARRANGEMENT OF A TRANSACTION TABLE

Another definitional problem to be examined is that of interindustry flows. Each x_{ij} represents an interindustry flow, i.e., the flow of goods from industry i to industry j . However, for $i=j$, i.e., x_{ii} , the term becomes an intraindustry flow. The term x_{22} represents the output of industry 2 used by industry 2. For example, the chemical industry produces polyhydric alcohols such as ethylene glycol and pentaerythritol, both of which are used in the manufacturing of many other chemicals. In some interindustry models, the term x_{ii} is considered zero and the corresponding changes are made to the input and output totals; this convention is not used in the present model.

Another point to remember is that although there is an equal number of rows and columns, the matrix is not symmetrical, i.e., $x_{ij} \neq x_{ji}$. This is reasonable, since there is no reason that the output of industry i to industry j should equal the output of industry j to industry i .

In the preceding discussion, the interindustry flows were called x_{ij} . They represent the input from industry i to industry j . In the equation $\frac{x_{ij}}{X_j} = a_{ij}$, a_{ij} is called the activity coefficient. It represents the input from industry i per unit of output from industry j . Therefore, the activity coefficient may be obtained by dividing each element in the transactions table by its column total. The preceding equation can be written as $x_{ij} = a_{ij}X_j$. The economist will recognize this as a linear homogeneous production function. Although activity coefficients may be calculated for all industries, only those in the endogenous sector (Quadrant II) are considered to be constant. This is a very important assumption and is discussed in detail in subsequent

chapters. The only reason for including it in the present discussion is to offer some meaningful criteria for distinguishing the endogenous sector from the exogenous sector, Quadrant I.

Quadrant III.-- As an aggregate, the elements in this quadrant approximate the value added by each industry. Since X_j represents the value of the total output and x_{ij} ($j=1 \dots n$) represents interindustry purchases, their difference is equal to the value added by manufacturing, i.e., the difference between the value of shipments and cost of materials, including other related costs such as containers and supplies. The entries in Quadrant III, with the exception of Government and Imports, are often called primary inputs since they usually give rise to profits, wages, and salaries.

Quadrant IV.-- The data in Quadrant IV are not used in the basic model or any of its applications. The only reason for including it is to illustrate the complete breakdown of interindustry transactions; in many transaction tables it is not listed. The entries in this quadrant represent the primary inputs to the final demand sectors. Although this information has little use in interindustry analysis, it is quite important in analyzing national income data.

Quadrant I.-- In the following chapters, the column entries in this quadrant comprise the exogenous sector of the model since they are external to the industries which form the transactions matrix. The industries in the transactions matrix, the first n columns and the

first n rows, are usually denoted as the endogenous sectors of the model since they are the only manufacturing industries in the transactions matrix. The exogenous sector is occasionally called the final demand or bill of goods. The endogenous sectors are often called the processing sectors. As stated in the discussion of Quadrant II, the activity coefficients for the final demand sector are not considered to be constant, i.e., the input to the industry is not a linear function of its output.

The elements in the exogenous sector of the model are often called "autonomous" elements to emphasize their independency from the endogenous sector. They are independent since there is no predetermined output from a given input. An excellent example of a final demand sector is the government entry. The column titled Government indicates that portion of an industry output which is purchased by the government. Since the actions of the government do not depend on any technical process, its activity coefficient is not considered to be constant. The government's purchase of the output of any industry is influenced by political, social, and financial pressures. However, in the endogenous sectors these conditions do not exist. For example, the input of steel to the automobile industry is more likely to be a constant percent of its output than will be governmental expenditures for the products of the steel industry.

A Mathematical Statement of the Model

Consider a transactions table with four industries in the

endogenous sector. The final demand is considered as one sector and is expressed as an aggregate value. The following equations are obtained by adding all of the elements in each row:

$$\begin{array}{l}
 x_{11} + x_{12} + x_{13} + x_{14} + Y_1 = X_1 \\
 x_{21} + x_{22} + x_{23} + x_{24} + Y_2 = X_2 \\
 x_{31} + x_{32} + x_{33} + x_{34} + Y_3 = X_3 \\
 x_{41} + x_{42} + x_{43} + x_{44} + Y_4 = X_4
 \end{array}$$

Where:

x_{ij} = the output from industry i to industry j

Y_i = the final demand for the products of industry i

X_i = gross output of industry i

One of the primary objectives of input-output analysis is to determine the output required from each industry given the final demand for its products, i.e., given Y_1 , Y_2 , Y_3 , and Y_4 , determine X_1 , X_2 , X_3 , and X_4 . An examination of the equation system will reveal the presence of twenty unknowns and four equations.

Assuming that the equations are independent and consistent, sixteen additional equations are needed in order to obtain a unique solution. These additional equations may be obtained from the production function for each industry.

$$\text{Since } a_{ij} = \frac{x_{ij}}{X_j}$$

$$x_{ij} = a_{ij}X_j$$

Then

$$\begin{aligned}
 & x_{11} = a_{11}X_1; \quad x_{12} = a_{12}X_2; \quad x_{13} = a_{13}X_3; \quad x_{14} = a_{14}X_4 \\
 & x_{21} = a_{21}X_1; \quad x_{22} = a_{22}X_2; \quad x_{23} = a_{23}X_3; \quad x_{24} = a_{24}X_4 \\
 & x_{31} = a_{31}X_1; \quad x_{32} = a_{32}X_2; \quad x_{33} = a_{33}X_3; \quad x_{34} = a_{34}X_4 \\
 & x_{41} = a_{41}X_1; \quad x_{42} = a_{42}X_2; \quad x_{43} = a_{43}X_3; \quad x_{44} = a_{44}X_4
 \end{aligned}$$

II

Substituting equation Set II into equation Set I,

$$\begin{aligned}
 & a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{14}X_4 + Y_1 = X_1 \\
 & a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + a_{24}X_4 + Y_2 = X_2 \\
 & a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + a_{34}X_4 + Y_3 = X_3 \\
 & a_{41}X_1 + a_{42}X_2 + a_{43}X_3 + a_{44}X_4 + Y_4 = X_4
 \end{aligned}$$

III

Equation Set III may be written as:

$$\begin{aligned}
 & (1-a_{11})X_1 - a_{12}X_2 - a_{13}X_3 - a_{14}X_4 = Y_1 \\
 & -a_{21}X_1 + (1-a_{22})X_2 - a_{23}X_3 - a_{24}X_4 = Y_2 \\
 & -a_{31}X_1 - a_{32}X_2 + (1-a_{33})X_3 - a_{34}X_4 = Y_3 \\
 & -a_{41}X_1 - a_{42}X_2 - a_{43}X_3 + (1-a_{44})X_4 = Y_4
 \end{aligned}$$

IV

Define, a , as the matrix formed by the activity coefficients

$$a = \begin{bmatrix} (1-a_{11}) & -a_{12} & -a_{13} & -a_{14} \\ -a_{21} & (1-a_{22}) & -a_{23} & -a_{24} \\ -a_{31} & -a_{32} & (1-a_{33}) & -a_{34} \\ -a_{41} & -a_{42} & -a_{43} & (1-a_{44}) \end{bmatrix}$$

Let:

I equal the identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Y equal the column vector of final demands

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}$$

X equal the column vector of outputs

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Therefore in matrix form equation Set IV may be written as:

$$(I-a) X = Y$$

$$(2.3-1) \quad X = (I-a)^{-1}Y$$

Equation (2.3-1) is the input-output model. Given a vector Y representing the final demand for each industry and the inverse of (I-a), the output of each industry is obtained from its product. Equation (2.3-1) applies to a n by n matrix and is not restricted to the four-industry example.

This section has discussed the elementary properties of the

model. No attempt has been made to discuss the static, dynamic, open, closed, or other important properties of the model. These topics, in addition to other applications of the basic model, are discussed in the Appendix.

CHAPTER 3

THE PROCEDURE FOR THE CONSTRUCTION OF AN INPUT-OUTPUT TABLE

3.1 Introduction

Basic to the application of any interindustry model is the construction of an input-output table. This chapter discusses the methodology involved in the construction of such a table; and the next chapter contains a numerical example of the procedures outlined in this chapter. The data in an input-output table are classified into one of three general sectors: endogenous (interindustry flows), exogenous (final demand), and gross output. Therefore, the procedures described in this chapter begin with an estimation of gross output and then proceed to a discussion of the methods involved in estimating the endogenous and exogenous flows. One of the primary objectives of this thesis is to discuss the applications of the interindustry table to market analysis; therefore this chapter contains a description of the methodology involved in converting gross outputs to employment and income data for a given region.

Exclusive Use of Secondary Data Sources

The input-output table is constructed from secondary data. Primary data need no transformation and may serve as input to the model in

its existing form; secondary data must be modified before they can serve as input to the model. Secondary data are modified by applying specific transformations to them with the hope that the transformed data will be the same as primary data. Unfortunately the majority of the transformations are not 100 percent efficient, i.e., the modified secondary data are not identical to the primary data.

Obviously, primary data are used when they are available or when they may be obtained at a reasonable cost. When this condition does not exist, secondary data are substituted for primary data. In the present study, the principle reason for using secondary data in lieu of primary data is the element of cost. The required primary data would have to be acquired by interviewing each firm in the sample and then aggregating the data by industry type. This operation is very expensive since the interviewer may have to spend several days with each firm in order to receive accurate responses in the questionnaire. For the numerical example in the next chapter the scope of such a study has been compared with that of similar studies in order to obtain a rough estimate of the cost involved in such an undertaking. As a result of this investigation, it is concluded that the anticipated study would cost \$25,000. The basic model was constructed from secondary sources since these funds are not available at the present time.

The substitution of secondary data for primary data is not a serious restraint on the analysis of interindustry models since the major objective of this study is to examine the methodology involved in analyzing the results obtained from an interindustry model. The

methodology involved in the analysis of the results of the model are perfectly general and independent of the data sources. Therefore, the methodology employed in this thesis is unique in that the entire input-output table was constructed from secondary data, which eliminated the need for a costly questionnaire survey.

Limitations on the Use of Secondary Data Sources

From the previous discussion it is evident that in general the use of primary data obtained from a survey is preferable to data obtained from secondary data sources. However, the use of primary data does not insure a greater accuracy in the results generated by the model than does the use of secondary data. The use of data obtained by the use of a poorly designed questionnaire or an unrepresentative sample may introduce greater error into the results than does the use of secondary data. There are several reasons for this phenomenon. First, the major sources of secondary data are governmental agencies who have the resources and experience needed for an accurate collection of data. Usually those engaged in local regional studies are limited to the amount of resources available for data collection and therefore, cannot be as exhaustive in their analysis as comparable studies conducted by governmental agencies. Second, local regional studies are usually conducted by small interindustry groups who have to supervise many steps of the construction of the model. Therefore, the data collection procedures may be subjected to various inaccuracies resulting from a lack of experience and the improper supervision of field personnel by the interindustry group.

However, if the sample is chosen with a reasonable degree of accuracy, an input-output table constructed from primary data is far superior to that constructed from secondary data. Primary data are obtained by a survey of the firms within the region; secondary data usually apply to some area larger than the region under consideration and must be scaled down so as to apply to the particular region for which the model is being constructed. The disadvantage in constructing an input-output table based on primary data is the time and expense involved in the collection and aggregation of the information obtained from the field survey.

The most serious limitation in the substitution of secondary data for primary data is that the scope of the inferences based on the output of the model may be limited. This presents no restrictions on the methodology but it does limit the applicability of the results for the solution of specific problems.

The Advantages in Using Secondary Data Sources

One of the greatest advantages in the use of secondary data as a substitute for primary data is that it permits an individual or a very small interindustry group to construct a preliminary model which does not require the vast quantity of resources demanded by a field survey.

Another advantage is that a pilot study based on the construction of a preliminary model using secondary sources is invaluable. A pilot study requires a thorough investigation of the existing sources

of data; the outcome of such an investigation permits a more intelligent design of the questionnaire to be used in the survey. A pilot study which is made previous to performing the survey will also pinpoint some of the major problems that the interindustry group are going to encounter. It is almost impossible to foresee all the problems which will occur by merely examining other interindustry studies. Depending on the objectives of the study, a pilot study may offer a solution to the given problem and thereby eliminate the survey in its entirety.

The Purpose of a Numerical Example

It is possible to describe the operation of the model without using a numerical example; however, there are several advantages in this approach. First, the use of a numerical example permits a detailed description of the methodology to be used in analyzing the results of the model. Second, the use of a numerical example, in addition to providing a basis for the description of the statistical methodology, also gives insight into the nature of empirical problems to be encountered. Without a numerical example it would be difficult to give an economic interpretation of the results generated by the model.

Therefore, there are two alternatives for obtaining the data for the model, namely the use of hypothetical data or secondary data. As previously stated, the methodology is general and would apply to both types of data. The advantage in using hypothetical data is that the

properties of the model can be examined without an extensive data search. The use of secondary data also has definite advantages. First, the results based on secondary input data will approximate the results that would be obtained from the use of primary data. Second, this information will permit the development of ideas concerning the interdependence of business activities in the given region before performing the survey. After making the appropriate transformations on the secondary data, the problems encountered in constructing the model and making extensions to it will be similar for both data sources. The data used in the construction of the model were obtained from an analysis of Duval County in the State of Florida. The purpose in obtaining these data was to overcome the criticism that the use of hypothetical data in a theoretical model restricts the scope of the analysis.

Although the transactions table is derived from national coefficients, it is quite valuable. In fact, many of the major input-output studies, such as Maryland, Utah, California, and Texas, have also used the national coefficients in the construction of regional models. An analysis which is based on the data obtained from the national coefficients will, in addition to illustrating the methodology to be used in the construction of an interindustry model, give some insight into the functioning of Duval County economy.

Fortunately Duval County is one of the Standard Metropolitan Statistical Areas utilized by the Bureau of the Census in the classification of statistical information. Therefore, the 1958 Census of Manufacturers¹ will be a major source of the data utilized in the construction of

¹U. S. Department of Commerce, Bureau of the Census. 1958 Census of Manufacturers: 1961. Area Statistics, Vol. III.

the input-output table. Volume III, Area Statistics, of the 1958 Census of Manufacturers² furnishes data for selected counties in Florida, in which Duval County statistics are included. Therefore, by combining the above sources of information, it is possible to obtain a relatively accurate account of the aggregate economic activity in Duval County.

The term "aggregate" is used to refer to the type of data furnished by the Census³ since it merely gives the value added by a given industry, not the distribution of a given industry's output to other industries within the complex. A knowledge of the distribution of a given industry's output to other industries in the complex would form the foundation for the construction of an input-output table from primary data sources. Nonetheless, the use of secondary data from Duval County Area will increase the realism in the model even though they are scaled down by the use of coefficients which are calculated on a national basis and are not entirely applicable to a specific region such as Duval County.

3.2. The Construction of the Input-Output Table

General Outline

The input-output (transactions) table is constructed in the following manner. First, the gross outputs of each industry in the model are estimated. Second, the sum of the endogenous flows are estimated by the use of national coefficients. Third, the exogenous flows

²Ibid.

³Ibid.

are calculated by subtracting the sums of the endogenous flows from the estimated output of each industry.

Using the notation employed in Section 2.3, the above outline may be summarized in the following manner:

1. Estimate X_j
2. Estimate x_{ij} by computing $a_{ij}X_j$, where a_{ij} is obtained from the 1947 table⁴
3. Calculate Y_i , $Y_i = X_i - \sum_j a_{ij}X_j$

Selection of Industries

Before making any estimates of industry output, it is mandatory to determine the dominant industries in Duval County. Dominant industries in a given area are those industries which contribute the most to employment and income in the given region.

The industries in this model are limited to the manufacturing sector since the government has published the activity coefficients for this sector. The 1958 Census of Manufacturers⁵ has a list of the major industries in Duval County. This source provided a means for selecting the industries to be considered in the model.

Estimation of Gross Outputs

In order to obtain a realistic estimate of the elements in the final demand sector, the corresponding value of shipments or gross output of each producing industry within the manufacturing sector must be

⁴W. D. Evans, and M. Hoffenberg. "The Interindustry Relations Study for 1947." The Review of Economics and Statistics, XXXIV (1952), 97-142.

⁵1958 Census of Manufacturers, p. 9-11.

calculated. The gross outputs of an industry refer to the total value of shipments at producer's prices.

Unfortunately, the Bureau of the Census in its publications does not furnish the value of shipments for each industry in the manufacturing sector. Mr. Maxwell R. Conklin, Chief of the Industry Division, Bureau of the Census, states in a private letter,

the value of shipments does not appear in the Census data because the figures for 3-digit⁶ and 2-digit industry groups would include a considerable amount of duplication since, in many industry groups, the shipments from establishments in a constituent 4-digit industry are often used as materials by another 4-digit industry within the same group. As a matter of policy the Bureau of the Census does not publish such summary values of shipment figures. Value added, which is the difference between cost of materials and value of shipment, is considered a more accurate measure of the economic contribution of establishments when figures are summarized to industry group totals.⁷

Therefore, the immediate problem is to estimate the value of shipments from the value added by manufacturing. The following discussion suggests three methods for determining the gross output of each industry in the model. Generally, there is no one method which always offers the most accurate estimate of gross output. The relative merits of each method are compared after all three of the methods have been described.

⁶The Standard Industrial Classification is an industrial classification of all activities into broad industrial divisions (manufacturing, mining, retailing, etc.). It further subdivides each division into major industry groups (two-digit industries) then into industry groups (three-digit industries) and finally into detailed industries (four-digit industries).

⁷Letter from Maxwell R. Conklin, Chief, Industry Division, Bureau of the Census, Washington, D.C., April 2, 1963.

Method I.-- In this method, the value of shipments and its accompanying value added by manufacturing are combined in order to obtain an average ratio of the value of shipments to the value added by manufacturing for selected industries within a given industry aggregate. Although the Bureau of the Census does not publish the total value of shipments for each two-digit industry, it does so for the majority of the three-digit industries. The difference between a two-digit and three-digit industry is that the former is the aggregate of all three-digit industries.

The above ratios must be calculated for the State of Florida, and not Duval County since the Area Statistics list only the value added by manufacturing for the three-digit industries. Therefore, there is the additional problem of converting the estimate of the state value of shipments to a regional value of shipments. The entire conversion of state value added by manufacturing to a regional value of shipments may be accomplished in the following manner:

1. For each three-digit industry within a given two-digit industry, compute the ratio of the state value of shipments to the state value added by manufacturing, and then calculate the average of the ratios for each two-digit industry.
2. The second ratio is calculated by dividing the number of employees in each industry within the given complex by the number of employees within the state who are in the industry classification.
3. Multiply the results of operation one by the results of operation two, and then multiply this product by the value added for each two-digit industry. The value added for the two-digit industry is based on that industry's total for the state, not the value added by that industry in Duval County. This final product will give an estimation of the value of shipments for each industry in Duval County.

General Formula:

$$V_j = \left(\frac{V_s}{V_m} \right) \bar{s} V_{ms} \left(\frac{E_j}{E_s} \right)$$

Where:

V_j = Gross output of jth industry in Duval County.

$\left(\frac{V_s}{V_m} \right) \bar{s}$ = The average of the ratio of the value of shipments to the value added by manufacturing for all three-digit industries within the state.

V_{ms} = Value added by manufacturing for each two-digit industry within the state.

E_j = Number of employees in the jth industry in Duval County.

E_s = Number of employees in the jth industry in the State of Florida.

Method II.-- Method II is merely an extension of Method I in that the ratio of the value of shipments to the value added by manufacturing based on state data, is applied directly to the value added by manufacturing for each industry within Duval County. The difference between Method I and Method II is that the former worked with the value added by manufacturing in the state, and scaled it down to Duval County by utilizing a constant ratio equal to the number of employees in the state divided by the number of employees working in Duval County. In Method II, the value added by manufacturing was not scaled down since it is the value added by the given industry in Duval County.

General Formula:

$$V_j = \left(\frac{V_s}{V_m} \right) \bar{s} (V_{mc})$$

Where:

V_j = Gross output of the j th industry in Duval County.

$\left(\frac{V_s}{V_m} \right) \bar{s}$ = The average of the ratio of the value of shipments to the value added by manufacturing for all three-digit industries within the state.

V_{mc} = Value added by manufacturing for each two-digit industry within the county.

Method III.-- Previously, the values of shipments for the sectors under consideration have been calculated by developing a general relationship between value added in a given industry and the corresponding value of shipments. However, there is another method of accomplishing the same task, namely, the use of industry productivity data. It is possible to obtain accurate data regarding the number of workers in Duval County in 1958. The average productivity of the workers in a given industry based on the national data of the years 1947 and 1950 has been calculated. Therefore, in order to estimate the value of shipments by Method III one merely multiplies the average productivity by the number of employees.

General Formula:

$$V_j = (\bar{P})(N)R$$

Where:

V_j = Gross output of the j th industry in Duval County.

\bar{P} = Average productivity of the employees in a given industry.

N = Number of employees.

R = Ratio of price index for the current year to the base year.

Method III assumes that the average productivity of the workers in each industry is constant before multiplying by the ratio of the price indexes. This assumption should not be alarming since a similar assumption was made in assuming that the activity coefficients in 1947 are applicable in 1962. Therefore, the assumption of constant productivity gives more consistency to the analysis even though it may widen the gap from the 1962 data.

A Comparison of Three Methods

As previously stated, no one of the three methods is always more accurate than the remaining two. However, of the three methods described, Method III is probably the most accurate in calculating the total value of shipments for the major industries in Duval County. The reason for this assertion is that one of the two components in the determination of the value of shipments is known quite accurately, i.e., the number of employees by industry in the manufacturing sector in Duval County. The inaccuracies in this approach are found in the productivity term since the average productivity of each worker in each manufacturing sector in Duval County can be calculated only from secondary sources. The problems involved in obtaining the average productivity for each industry are described in detail when output data are converted to employment data.

In the construction of the Duval County Input-Output Table, Methods I and II were not used for several reasons. First, there was a large discrepancy between the ratio of the value of shipments to the value added by manufacturing for the three-digit industries within each two-digit industry classification. Second, there were not sufficient data to provide an adequate weighting of three-digit industry output in order to reconcile the aforementioned discrepancies. Third, in many cases, the number of the three-digit industries for which output data were furnished, represented a small portion of the total number of three-digit industries.

Estimating Endogenous Flows

The empirical problems involved in the estimation of the endogenous flows are greatly reduced by the use of national activity coefficients. Given the outputs of each industry, i.e., the values obtained by the estimation procedures described in the previous section, the endogenous flows are calculated quite easily. The endogenous flows for each cell of the transactions table are obtained by multiplying its coefficient times the output of the industry, i.e., $x_{ij} = a_{ij} X_j$.

Estimation of Final Demand

After estimating the gross outputs and the endogenous flows of each sector in the model, the next step is to allocate this aggregate figure to the various input industries. The current problem is to distribute that amount which is equal to the difference between the total value of shipments and the inputs into the endogenous sector

among the competing sectors in the final demand sector, that is, the exogenous sector.

The mere fact that a given industry is included in the exogenous sector implies that its activity coefficients are not to be considered constant. However, the allocation of this total amount among the competing industries must be made with some accuracy if any economic inferences are to be made from the results. As previously stated, the methodology used in interpreting the results of the model is quite general and independent of the magnitude of the data utilized in its construction.

The allocation of the gross output over the endogenous sectors is solved by assuming that the national activity coefficients apply to Duval County. The current problem is to allocate the aggregate final demand figure over its component parts; three different approaches were used. The first approach is one in which the aggregate final demand figure was not broken down into its component parts. In this method the national coefficients are applied to the estimates of gross output for each sector in the model. By using these coefficients, the sum of the flows in the endogenous sector are determined. The value of the exogenous flows is calculated by subtracting the sum of the endogenous flows from the gross output.

The second approach assumes a hypothetical breakdown of the final demand sector. The purpose in doing this is to illustrate the methodology involved in analyzing the contribution of each component to the aggregate levels of output, employment, and income.

In the third approach, the aggregate final demand figure was not distributed over its component parts. However, independent estimates were made of one segment of the final demand sector, namely, consumer expenditures by industry type. The results of this analysis relate consumer expenditures to industrial output, employment, and income.

3.3 The Conversion of Total Output to Employment and Income Data

Employment Data

The purpose of this section is to describe the mechanics involved in converting output levels to employment data. The conversion of output to employment is one of the important extensions of the model. Although a knowledge of the output required to produce the final bill of goods is quite important to those involved in marketing, finance, and regional planning, a knowledge of the employment required to produce this output is equally important.

In order to make the conversion from output to employment figures, the level of output corresponding to a given bill of goods is divided by the output for each worker. Hence,

$$\text{Output per worker} = \frac{\text{Total output for industry}}{\text{Total employment (including the clerical workers within the industry)}}$$

Therefore:

$$\text{Total employment} = \frac{\text{Total output}}{\text{Output per worker}}$$

Output per worker may be calculated in several ways. Preferably, the total output of each industry in Duval County would be divided by its number of employees. Although the denominator of this fraction is known quite accurately, the numerator must be estimated. Since the numerator is estimated by making transformations on the value added by manufacturing, there might be considerable error in obtaining any output per worker from these data. For this reason the productivity calculations are based on national statistics which give accurate estimates of both output and employment for a given year.

Total output by industry, on a national basis, is given in the 1947 table.⁸ Therefore, the productivity for the workers in a given industry is calculated by dividing these industry outputs by the number of employees given in the 1950 Census.⁹ In all subsequent operations concerning the conversion of output to employment figures, the output level of each industry is divided by its average productivity in order to obtain the number of employees corresponding to that level of output.

Income Data

Thus far the analysis has been discussed in the following sequence: given a bill of goods, the required output was calculated; knowing the output per worker the level of employment induced by the change in output was determined. The induced employment due to the increase in output can be related to income figures by having a knowledge of the average income of the workers in each industry.

⁸Evans and Hoffenberg, *passim*.

⁹U.S. Department of Commerce, Bureau of the Census. U. S. Summary, U. S. Census of Population: 1950.

In subsequent discussions in which employment data are converted to income figures, the following transformations are used:

$$\text{Industry Income} = (\text{Number of workers}) (\text{Average income for each worker})$$

CHAPTER 4

EMPIRICAL RESULTS

This chapter contains the empirical results obtained by applying the statistical procedures described in Chapter 3 to the secondary data of Duval County, Florida. Generally, no additional comments or restatements of the information contained in Chapter 3 are made in this chapter.

4.1 Basic Data for the Construction of an Input-Output Table

Major Industries in Duval County, Florida

TABLE 4.1-1--MAJOR INDUSTRIES IN DUVAL COUNTY, FLORIDA

Industry	Number of Establishments	Number of Employees	Value added by manufacturing (\$1,000)
Food	88	4,195	49,677
Lumber	63	1,107	5,691
Furniture	38	682	4,835
Paper	19	1,909	29,142
Printing	71	1,560	11,630
Chemicals	30	1,093	12,947
Stone, Clay, and Glass	27	1,227	12,407
Fabricated Metal	59	1,144	10,500
Transportation	23	2,746	15,773
Eating and Drinking	583	1,341 ^a	31,988 ^b

Source: U S. Department of Commerce, Bureau of the Census. 1958 Census of Manufacturers: 1961. Area Statistics, Vol. III. p. 9-11.

^aMale Employees

^bReceipts(\$1,000)

Gross Outputs of the Major Industries

TABLE 4.1-2--GROSS OUTPUTS OF THE MAJOR INDUSTRIES

Industry	Average ^a Productivity (Dollars)	Number of ^b Employees	Output 1947 (Dollars)	Output 1962 ^c (Dollars)
Food	26,859	4,195	112,673,505	149,855,762
Lumber	6,975	1,107	7,721,325	10,269,362
Furniture	8,757	682	5,972,274	7,943,124
Paper	16,937	1,909	32,332,733	43,002,534
Printing	7,506	1,560	11,709,360	15,573,448
Chemicals	21,317	1,093	23,299,481	30,988,309
Stone, Clay, and Glass	10,487	1,227	12,867,549	17,113,840
Fabricated Metal	2,762	1,144	3,159,728	4,202,438
Transportation	16,397	2,746	45,026,162	59,884,795
Eating and Drinking ^d	7,870	1,341	10,553,670	14,036,380

Source: U. S. Department of Commerce, Bureau of the Census.
Statistical Abstract of the United States: 1962. Table 465.

^aTable 4.4-1

^bTable 4.1-1

^cRatio of price index for 1962 to price index for 1947 is 1.33.

^dThe output of this sector applies only to male help.

Estimation of Endogenous Flows

Activity Coefficients.-- Activity coefficients represent the value of input used per dollar of output. The national activity coefficients were applied to Duval County. The activity coefficients are the basic elements in the estimation of endogenous flows since the product of an activity coefficient and its corresponding industry

output, gives an estimation of interindustry flows, i.e., $x_{ij} = a_{ij}X_j$. The activity coefficients for the major industries in Duval County are given in Table 4.1-3. From Table 4.1-3 one can determine the amount of goods required from each industry in the model in order to produce \$1.00 of output. For example, it is observed that the Chemical industry must purchase the following amounts of goods to produce \$1.00 of Chemical products: 4.9 cents from the Food industry, .3 cents from the Lumber industry, nothing or a negligible amount from the Furniture industry, 2.4 cents from the Paper industry, .1 cent from the Printing industry, 19.1 cents internally, 1.9 cents from the Stone, Clay and Glass industry, and nothing or a negligible amount from the Fabricated Metal, Transportation, and Eating and Drinking industries. The inputs from the exogenous sectors of the model per dollar of chemical output are not calculated since the activity coefficients in the exogenous sectors are not considered to be constant.

Example of the Calculation of Endogenous Flows.-- From the general formula,

$$x_{ij} = a_{ij}X_j$$

it follows that:

$$(4.1-1) \quad x_{11} = a_{11}X_1$$

$$(4.1-2) \quad x_{74} = a_{74}X_4$$

Equation (4.1-1) states that the output of the Food industry going to the Food industry is equal to the appropriate activity coefficients times the output of the Food industry. $(.131870)(\$149,855,262) = \$19,762,006$.

TABLE 4.1-3-- ACTIVITY COEFFICIENTS

Industry	Food	Lumber	Furniture	Paper	Printing
Food	.131870	.000057	.000125	.003804	.000053
Lumber	.002174	.182243	.133882	.039158	.000094
Furniture	.000000	.000000	.002433	.000725	.000000
Paper	.012165	.000758	.005253	.332404	.168256
Printing	.001052	.000000	.000000	.000000	.119509
Chemical	.038979	.004359	.021779	.023454	.015073
Stone, Clay, Glass	.006784	.002398	.011905	.003641	.000000
Fabricated Metal	.000000	.000000	.001722	.000000	.000000
Transportation	.000067	.001560	.000000	.000000	.000000
Eating and Drinking	.000000	.000000	.000000	.000000	.000311

Industry	Chemical	Stone, Clay and Glass	Fabricated Metal	Transportation	Eating and Drinking
Food	.049224	.000408	.000000	.000000	.261397
Lumber	.003232	.003518	.002205	.003263	.000361
Furniture	.000000	.000000	.000000	.001805	.000000
Paper	.023798	.037024	.002194	.002311	.004261
Printing	.001132	.000000	.000000	.000000	.002210
Chemical	.190897	.023855	.007595	.007774	.003196
Stone, Clay, Glass	.018544	.088765	.002608	.013493	.004410
Fabricated Metal	.000000	.000000	.017858	.000678	.000000
Transportation	.000000	.000156	.000000	.308642	.000105
Eating and Drinking	.000000	.000000	.000000	.000000	.040390

Source: W. D. Evans, and M. Hoffenberg, "The Interindustry Relations Study for 1947." The Review of Economics and Statistics, XXXIV, (1952), 97-142.

Similarly, equation (4.1-2) states that the output of the Stone, Clay, and Glass industry which goes to the Paper industry is equal to the appropriate activity coefficient times the output of the Paper industry, $(.003641)(\$43,002,534) = \$156,672$. A repetition of this procedure for all industries in the model will lead to the estimation of all the exogenous flows in the model.

Estimation of Final Demand

Final demand estimates are obtained from an algebraic solution of the basic model.

$$\sum_{j=1}^n x_{ij} + Y_i = X_i$$

$$Y_i = X_i - \sum_{j=1}^n x_{ij}$$

For example, consider the four industry examples in Chapter 2; the final demand for industry 1 is given by

$$(4.1-3) \quad Y_1 = X_1 - x_{11} - x_{12} - x_{13} - x_{14}$$

Equation (4.1-3) states that the final demand for industry 1 is equal to the total output of industry 1 less all interindustry (endogenous) flows.

A Numerical Example for the Duval County Model.-- For the Food industry

$$X_1 = \$149,855,762 \quad (\text{Table 4.1-2})$$

The interindustry (endogenous) flows for the Food industry were calculated as described in the previous section. The following

interindustry flows (dollars) are a result of these calculations:

$$x_{11} = \$19,762,006$$

$$x_{12} = 585$$

$$x_{13} = 1,998$$

$$x_{14} = 163,581$$

$$x_{15} = 825$$

$$x_{16} = 1,525,168$$

$$x_{17} = 6,982$$

$$x_{18} = 0$$

$$x_{19} = 0$$

$$x_{20} = 366,906$$

Therefore,

$$Y_1 = 149,855,262 - (19,762,006 + 585 + 1,998 + 163,581 + 825 + 1,525,168 + 6,982 + 366,906) = 128,028,516$$

A similar calculation for each industry in the table will give the final demand for each industry in the model.

4.2 The Input-Output Table

The empirical results of Section 4.1 may be summarized in a table commonly called an input-output or transactions table. The construction of an input-output table, without any additional modifications or extensions, gives an insight into the functioning of a given complex. In essence, it is a method for summarizing interindustry flows in a concise logical manner. The use of these interindustry flows in market analysis is described in a following chapter. Table 4.2-1 is the input-output table for Duval County.

The table is constructed by filling either n rows or n columns. Each row total represents the sales of that given industry; each column total represents the purchases of the industry, i.e., every sale is a purchase. This symmetry in the flow table gives rise to the name input-output table. By examining the fifth row in Table 4.2-1, the Printing industry has a total output of sales of \$15,573,448 of which \$35,078 was purchased by the Chemical industry. Likewise by examining the sixth column it is observed that the Chemical industry purchased \$35,078 of materials from the Printing industry.

The Row Entries

Each row total represents the total output or sales of the industry, valued at producer's prices. For example, reading across the fourth row, the Paper industry sold \$1,823,044 to the Food industry, \$7,786 to the Lumber industry, and \$5,980 to Eating and Drinking places. Out of a total output of \$43,002,534, \$22,690,764 is sold in the exogenous or final demand sector of the economy. The total sold in the endogenous sector for any given industry is equal to the difference between total output and the total value of exogenous flows (final demand) i.e., \$20,311,770.

The final demand figure is made of many elements, In this model, the following sectors: Government, Inventory Change, Gross Private Capital Formation, Construction, Households, and Imports comprise the aggregate value of exogenous flows. This breakdown is quite arbitrary and may change from model to model depending on the available data and the objective of the analysis. The present discussion is concerned

TABLE 4.2-1--INTERINDUSTRY FLOWS
(Dollars)

Industry	Food	Lumber	Furniture	Paper	Printing	Chemicals
Food	19,762,006	585	1,998	163,581	825	1,525,168
Lumber	325,799	1,871,701	1,063,440	1,683,893	1,463	100,157
Furniture	--	--	19,328	31,176	--	--
Paper	1,823,044	7,786	41,725	14,294,214	2,620,326	737,459
Printing	157,652	--	--	--	1,861,286	35,078
Chemicals	5,841,305	44,768	172,993	1,008,584	234,738	5,915,575
Stone, Clay and Glass	1,016,625	24,628	94,562	156,572	--	574,647
Fabricated Metal	--	--	13,679	--	--	--
Transportation	10,040	1,602	--	--	--	--
Eating and Drinking	--	--	--	--	48,430	--
Industry	Stone, Clay and Glass	Fabricated Metal	Transportation	Eating and Drinking	Final Demand	Total Output
Food	6,982	--	--	366,906	128,028,516	149,855,762
Lumber	60,210	9,266	195,404	506	4,957,523	10,269,362
Furniture	--	--	108,092	--	7,784,528	7,943,124
Paper	633,623	9,220	138,393	5,980	22,690,764	43,002,534
Printing	--	--	--	3,102	13,516,330	15,573,448
Chemicals	408,274	31,917	465,544	4,486	16,860,128	30,988,309
Stone, Clay and Glass	1,519,110	10,959	808,025	6,190	12,902,519	17,113,840
Fabricated Metal	--	75,047	40,601	--	4,073,111	4,202,438
Transportation	2,669	--	18,482,962	147	41,387,522	59,884,795
Eating and Drinking	--	--	--	566,920	13,421,030	14,036,380

Source: Tables 4.1-2 and 4.1-3.

with the aggregate values of exogenous flows but eventually this figure is allocated over its components so as to give an example of the methodology used in extending the model and interpreting the data therefrom.

The Column Entries

Constructing an input-output table from column entries would be identical to one constructed from data based on row entries; however, there may be a greater difficulty in obtaining the data for the column entries. The accounting procedures of many firms are so devised that they have an accurate record of their expenditures and only a limited knowledge of the sources of their receipts. Therefore it is quite probable that there will not be enough data available for the construction of the table based on column entries. The omission of row or column entries in the exogenous sector is not of primary importance since the inverse of the matrix is independent of them.

Consider the first column, Food. The first entry \$19,762,006 indicates that the Food industry purchased this amount from themselves, probably in the form of basic meat and dairy products before they were processed. The Food industry also purchased \$5,841,305 from the Chemical industry in the form of preservatives and processing agents. If the final demand were broken down into component parts, the table should have six additional rows and columns titled: Government, Inventory Change, Gross Capital Formation, Households, Construction, and Imports. However, due to the lack of adequate data for these cells, they are not included in the present analysis. The breakdown of the final demand sector is discussed in Chapter 5.

4.3 Interdependence Coefficients

The interdependence coefficients for the model are shown in Table 4.3-1. They are obtained by inverting the matrix formed by the difference between the identity matrix and the matrix represented by the activity coefficients in Table 4.2-1. The matrix was inverted on an IBM 709 electronic computer located in the Computing Center at the University of Florida. The mathematical properties of this matrix and its inverse are discussed in Chapter 2 and are not repeated in this section. The present discussion involves the economic interpretations of the inverse.

The entries in Table 4.3-1 are called interdependence coefficients in order to illustrate the dependency of one sector on the output of another sector. Table 4.1-3 lists the activity coefficients which show the direct purchases required per dollar of output. This table shows no relationship among the indirect flows in the various sectors. For example, assume that a market research group is hired by some industrial complex to make projections concerning the demand by consumers for the output of each industry in the model, i.e., to make an estimate of consumer expenditures which are in essence the entries in the column titled Households. Assume the following estimates of household expenditures by industry type: Food, 40 million; Lumber, 1 million; Furniture, 4 million; Paper, 5 million; Printing, 3 million; Chemical, 2 million; Stone, Clay and Glass, 1 million; Fabricated Metal, 5 million; Transportation, 10 million; and expenditures at Eating and Drinking places, 1.5 million.

TABLE 4.3-1--INTERDEPENDENCE COEFFICIENTS

Industry	Food	Lumber	Furniture	Paper	Printing
Food	1.16173	.00551	.00145	.08574	.01662
Lumber	.00597	1.22320	.16560	.00843	.00277
Furniture	.00002	--	1.00246	.00001	--
Paper	.02417	.00183	.00993	1.50019	.28749
Printing	.00177	--	--	.00192	1.13610
Chemicals	.05711	.00677	.02868	.04456	.02978
Stone, Clay and Glass	.02400	.00505	.01394	.01306	.00257
Fabricated Metal	--	--	.00176	--	--
Transportation	.00012	.00028	.00004	.00002	--
Eating and Drinking	--	--	--	--	.00037
	1.27489	1.24264	1.22386	1.65393	1.47570
Industry	Chemicals	Stone, Clay and Glass	Fabricated Metal	Transportation	Eating and Drinking
Food	.00306	.00411	.00024	.00043	.31691
Lumber	.05984	.00664	.00325	.00704	.00235
Furniture	--	.00080	--	.00263	.00001
Paper	.04750	.06222	.00389	.00680	.01435
Printing	.00006	.00008	.00010	.00001	.00311
Chemicals	1.24159	.03437	.00980	.01490	.02012
Stone, Clay, and Glass	.00392	1.09810	.00299	.02158	.01166
Fabricated Metal	--	--	1.01918	--	--
Transportation	.00001	.00025	--	1.44643	.00019
Eating and Drinking	--	--	--	--	1.04210
	1.35598	1.20657	1.03845	1.49982	1.41080

Source: Table 4.1-3

Multiplying this final bill of goods, i.e., the column representing consumer expenditures, by the elements in the appropriate column of Table 4.1-3 the direct inputs required from each industry in order to produce the estimated consumer demand are determined. The multiplication of the elements in the Food column by 40 million will determine the inputs required from each industry to produce the \$40 million of food products. Likewise, multiplying the Lumber column by 1 million will determine the inputs to the Lumber industry from the other industries in the complex in order that the Lumber industry manufacture or produce \$1 million of output. A repetition of this process for every column will yield the direct inputs required to furnish the given bill of goods. The inputs obtained from this process are called first-round requirements.

Assume that the Chemical industry must produce an output of \$10 million in order that the bill of goods previously discussed be produced. This \$10 million output of the Chemical industry which serves as inputs to other industries is the direct output required to meet the bill of goods desired by Households. However, no consideration has been given to the indirect requirements, i.e., in order that the Chemical industry produce an output of \$10 million, it must have a corresponding inflow of material. These induced inputs arising from direct inputs related to the given bill of goods are called indirect purchases, or in this case second-round requirements.

In order to determine the necessary inputs to the Chemical industry, the elements in the Chemical column are multiplied by 10 million.

This procedure is repeated for all sectors in the model, i.e., the first-round requirements are multiplied by the entries in each column to obtain the second-round requirements. Theoretically, this process could be continued until an infinite number of rounds have been computed. Fortunately, the magnitude of the output begins to converge as the number of rounds increases, i.e., the addition to input of the $n + 1$ round is so small that the calculation may be terminated after completing a finite number of iterations. This entire iterative procedure may be accomplished in one step by inverting the matrix formed by the activity coefficients.

Therefore, the matrix in Table 4.3-1 is unique in that it gives both the direct and indirect requirements that must be produced by the row sectors to deliver one dollar of final demand by the column sectors. Consider the Furniture industry, reading down column three and rounding to one-tenth of a cent, that for each dollar of final demand for Furniture the Food industry must produce .1 cents of output, the Lumber industry must produce 16.6 cents of output, the Furniture industry must produce 100.2 cents of output (one dollar of final demand plus .2 cents from itself) etc., down the column. The rows and columns of the final demand sector do not enter into the analysis since they are in the exogenous sector.

4.4 Supplementary Transformation Functions

Although the basic model gives an estimation of the output for each industry in the model, it does not offer a direct estimate of the levels of employment and income corresponding to a given level of output. The conversion of output data to income and employment data necessitates

the use of conversion techniques that are not found in the basic model, i.e., supplementary transformation functions.

The use of these transformation functions is discussed in Section 3.3 and is not repeated at this time. Basic to these transformations are two ratios for each industry in the model, namely, output per worker and income per worker. This information for the major industries in Duval County is summarized in Tables 4.4-1 and 4.4-2.

TABLE 4.4-1--OUTPUT PER WORKER BY INDUSTRY FOR DUVAL COUNTY

Industry	<u>1947 Industry Output</u> <u>1950 Industry Employment</u>	1947 (Dollars)	1962 (Dollars)
Food	<u>37,636,000,000</u> 1,401,199	26,859	35,722
Lumber	<u>6,002,000,000</u> 860,512	6,975	9,277
Furniture	<u>2,892,000,000</u> 330,243	8,757	11,647
Paper	<u>7,899,000,000</u> 466,378	16,937	22,526
Printing	<u>6,447,000,000</u> 858,968	7,506	9,983
Chemicals	<u>14,050,000,000</u> 659,091	21,317	28,352
Stone, Clay, and Glass	<u>4,844,000,000</u> 461,887	10,487	13,948
Fabricated Metal	<u>2,316,000,000</u> 838,575	2,762	3,673
Transportation	<u>14,265,000,000</u> 869,974	16,397	21,808
Eating and Drinking	<u>13,270,000,000</u> 1,686,108	7,870	10,467

Source: W. D. Evans, and M. Hoffenberg, "The Interindustry Relations Study for 1947." The Review of Economics and Statistics, XXXIV 1952, 97-142. U. S. Department of Commerce, Bureau of the Census, U. S. Summary, U. S. Census of Population: 1950. Table 4.1-2.

TABLE 4.4-2--AVERAGE INCOME BY INDUSTRY FOR DUVAL COUNTY

Industry	Payroll 1958 (\$1,000)	Payroll 1962 (\$1,000)	Employment 1958	Income/Worker 1958 (Dollars)	Income/Worker 1962 (Dollars)
Food	15,416	16,957	4,195	3,670	4,037
Lumber	3,084	3,392	1,107	2,785	3,063
Furniture	2,450	2,695	682	3,592	3,952
Paper	10,302	11,332	1,909	5,396	5,935
Printing	7,486	8,234	1,560	4,798	5,277
Chemicals	4,530	4,983	1,093	4,145	4,559
Stone, Clay, and Glass	4,492	4,941	1,227	3,620	3,982
Fabricated Metals	5,076	5,583	1,144	4,437	4,880
Transportation	12,626	13,888	2,746	4,598	5,057
Eating and Drinking	2,139	2,353	1,341	1,596	1,755

Source: U. S. Department of Commerce, Bureau of the Budget, U. S. Census of Population: 1960, Florida. PC (1) - 11-D; Detailed Characteristics, p. 11-607; U. S. Department of Commerce, Bureau of the Census. 1958 Census of Manufacturers: 1961. Area Statistics, Vol. III. p. 9-11; U. S. Department of Commerce, Bureau of the Census. Statistical Abstract of the United States: 1962. p. 228.

CHAPTER 5

MARKETING APPLICATIONS OF INPUT-OUTPUT ANALYSIS

5.1 Introduction

The previous chapters contain an examination of the nature, role, and characteristics of input-output analysis as applied to marketing. Although the technical operation and derivation of input-output models have been discussed in considerable detail there have been no applications of input-output models to current marketing problems. The objective of this chapter is to provide numerical examples which illustrate the use of input-output models for the solution of marketing problems.

A survey of the literature in this area has revealed only a few articles which attempt to apply input-output techniques to marketing problems, the majority of which used a qualitative approach as opposed to the quantitative approach under consideration. The quantitative models in marketing are usually applied to advertising expenditures, pricing policies, and inventory problems, none of which has been solved by input-output analysis.

In Chapter 4, the basic components of the model have been developed, i.e., a table of interindustry flows, the activity coefficients, and the interdependence coefficients, which were obtained by inverting the matrix formed by subtracting the activity coefficients from an

identity matrix. The above components, which are designated as the basic model, give a great deal of information concerning the economic activity in a given complex. The components in the basic model contain a vast quantity of information and are used as a foundation for a variety of economic studies. The primary contribution of this chapter is contained in those modifications of the basic model which provide the market analyst with new tools for market research.

Thus far both national and area data have been used for the construction of the basic components of the model. The use of these data, in addition to illustrating the methodology involved in interpreting the results of an interindustry analysis, adds realism to the discussion. However, on occasion, hypothetical data are generated by making specific assumptions regarding the characteristics of the existing data. The end result of such assumptions is that the data used in some portions of the analysis may or may not resemble the actual data for Duval County. In any event, the assumptions are stated explicitly to prevent the reader from making incorrect inferences.

5.2 Modifications of the Basic Model

Forecasting Levels of Output, Employment, and Income

Assume that the county officials have hired an interindustry group to make a study of employment levels in the larger industries within the county. The objective of the study is to anticipate possible unemployment problems in the year 19xx so that appropriate actions may

be taken in order to minimize their effect on the community. Regional Planning Boards could use the information to forecast population changes since there is a tendency for workers to enter or leave the county depending on the level of employment. If there is an increase in employment, new workers need housing, utilities, and other public services. An advance knowledge of this information is an aid in efficient planning and provides the basis for an optimal expenditure of public funds.

TABLE 5.2-1--CURRENT LEVELS OF FINAL DEMAND, OUTPUT, AND EMPLOYMENT
BY INDUSTRY TYPE

Industry	Final Demand (Dollars)	Total Output (Dollars)	Employment
Food	128,028,516	149,855,762	4,195
Lumber	4,957,523	10,269,362	1,107
Furniture	7,784,528	7,943,124	682
Paper	22,690,764	43,002,534	1,909
Printing	13,516,330	15,573,448	1,560
Chemicals	16,860,128	30,988,309	1,093
Stone, Clay, and Glass	12,902,519	17,113,840	1,227
Fabricated Metal	4,073,111	4,202,438	1,144
Transportation	41,387,522	59,884,795	2,746
Eating and Drinking	13,421,030	14,036,380	1,341

Source: Tables 4.1-1 and 4.2-1.

After an exhaustive study of the economic conditions in the county, the interindustry group offers the following estimates of final demand for the basic industries in the area:

TABLE 5.2-2--HYPOTHETICAL FINAL DEMAND FOR THE YEAR 19xx BY INDUSTRY TYPE

Industry	Final Demand (19xx) (Dollars)
Food	82,879,528
Lumber	6,468,364
Furniture	7,903,703
Paper	17,698,811
Printing	13,674,827
Chemical	20,546,458
Stone, Clay, and Glass	14,571,083
Fabricated Metal	4,117,306
Transportation	41,373,298
Eating and Drinking	1,403,202

Although the 19xx final demand schedule is entirely hypothetical it was constructed to approximate the conditions in Duval County. One estimate (the Eating and Drinking sector) is unrealistic, the purpose being to check the linearity of the model in extreme cases. The output required to produce the estimated 19xx final demand was obtained by considering the final demand as a column vector and multiplying it by the interdependence coefficients in Table 4.3-1. In terms of matrix multiplication, the operation was of the following form:

A_{11}	A_{12}	A_{13}	\dots	A_{1n}	82879528
A_{21}	A_{22}	A_{23}	\dots	A_{2n}	6468364
.	7903703
.	17698811
.	13674827
.	20546458
.	14571083
.	4117306
.	41373298
A_{n1}	A_{n2}	A_{n3}	\dots	A_{nn}	1403202

where each A_{ij} represents an activity coefficient from Table 4.3-1.

Example:

$$(1.16173)(82879528) + (.00551)(6468364) + (.00145)(7903703) \\ + \dots (.31691)(1403202) = \$98,662,269.$$

The results of this multiplication are as follows:

TABLE 5.2-3--OUTPUT, EMPLOYMENT, AND INCOME BASED ON 19xx FINAL DEMAND

Industry	Output (Dollars)	Employment (19xx)	Income 19xx (\$1,000)
Food	98,662,269	2,761	11,146
Lumber	11,536,617	1,243	3,807
Furniture	8,045,446	690	2,726
Paper	34,776,551	1,543	9,157
Printing	15,724,073	1,575	8,311
Chemical	32,895,527	1,160	5,288
Stone, Clay, and Glass	19,400,421	1,390	5,535
Fabricated Metal	4,247,607	1,156	5,641
Transportation	59,859,945	2,744	13,876
Eating and Drinking	1,467,358	140	245

Source: Table 5.2-2.

The employment and income columns were obtained by making the appropriate conversions described in Chapter 3. A comparison of Tables 5.2-2 and 5.2-3 illustrates the advantage of using an input-output model in forecasting the effects of a change in output on the entire industry complex. For example, in the Food industry, the estimated final demand in 19xx represents a decrease of 35% from that of 1962 (consult Tables 5.2-1 and 5.2-2) i.e., $(\$128,028,516 - \$82,879,528) \div (\$128,028,516)$. The corresponding decrease in output was 34%, i.e.,

\$149,855,762 (Table 5.2-1) - \$98,662,269 (Table 5.2-3) ÷ \$149,855,762.

It is incorrect to assume that output will always decrease by an amount comparable to the change in final demand. Such an assertion is incorrect since inputs to one industry are sources of output from another industry, i.e., there is an interdependency in the complex. The final demand sector is not the only outlet for the output of an industry. Therefore, if the level of final demand is small compared to the magnitude of the endogenous flows there is no reason to believe that an x percent change in final demand will be accompanied by a corresponding change in output. For example, an examination of the change in the final demand for the Lumber industry reveals that a 30 percent increase in final demand results in only a 12 percent change in its output.

A change in final demand which affects the level of output of the industries in the complex has wide repercussions throughout the economy, the most obvious being a change in the level of employment brought on by a change in the level of output. Corresponding to the output and employment changes induced by a change in the final demand for the products of any given industry, there is also a change in the income received by the working group. Later in this chapter the use of this information in evaluating market potentials is discussed in conjunction with the output multiplier. The results of this analysis may be used in the solution of regional planning problems.

The Effect of Governmental Spending on Output, Employment, and Income

Chapter 4, which contains a discussion of the empirical results

obtained from the model, considered only an aggregate figure for the final demand sector. This section contains an analysis of one of the components of this aggregate figure, namely governmental spending; the next extension considers another component, household expenditures. In Table 4.2-1, the transactions table, the final demand figure was not distributed over its components due to a lack of adequate data. In a later modification of this model, the final demand figure is distributed into six categories: Construction, Government, Inventory Change, Gross Private Capital Formation, Households, and Exports (the corresponding row entry is Imports), the definitions of which are included in the Appendix. Unlike the data in the previous extension, no attempt was made to make the following hypothetical data resemble actual governmental expenditures in Duval County.

TABLE 5.2-4--HYPOTHETICAL GOVERNMENTAL EXPENDITURES BY INDUSTRY TYPE

Industry	Governmental Expenditures (Dollars)
Food	53,606,478
Lumber	79,690
Furniture	4,390,507
Paper	373,798
Printing	347,473
Chemical	3,164,154
Stone, Clay, and Glass	1,128,384
Fabricated Metal	28,857
Transportation	9,980,480
Eating and Drinking	1,404,604

The objective of this extension is to determine the role that governmental spending has on the complex whose interindustry flows are

given in Table 4.2-1. In this table, the output figures are stated for each industry as a whole. The current problem is to determine the contribution that governmental expenditures make to these total quantities. In order to obtain the solution to the problem, the column of governmental expenditures is considered as a column vector and is multiplied by the matrix of interdependence coefficients given in Table 4.3-1. The results of such an operation are shown in Table 5.2-5.

TABLE 5.2-5--THE GOVERNMENT'S CONTRIBUTION TO OUTPUT, EMPLOYMENT, AND INCOME

Industry	Output (Dollars)	Employment	Income (\$1,000)
Food	62,836,988	1,759	7,101
Lumber	1,427,663	154	471
Furniture	4,429,447	380	1,501
Paper	3,207,790	142	842
Printing	4,048,075	405	2,137
Chemical	7,452,579	263	1,119
Stone, Clay, and Glass	2,845,179	204	812
Fabricated Metal	44,063	12	59
Transportation	14,443,112	662	3,347
Eating and Drinking	1,465,035	140	245

Source: Table 5.2-4.

The employment and income figures were obtained from the output figures by the methods outlined in Chapter 3. Therefore, Table 5.2-5 gives the government's contribution to the levels of income and employment resulting from governmental expenditures in the given complex. This analysis may be extended by investigating the effects of a 10 percent increase in governmental spending on the levels of output, employment,

and income. This problem is solved by multiplying the new level of government expenditures by the matrix formed from the interdependence coefficients and then comparing the induced change with the original level of employment.

Projected Output Levels for a Given Schedule of Consumer Expenditures

Thus far, the modifications of the basic model involved aggregate projections, i.e., those pertaining to all sectors in the complex. At this time, the model is used to determine effects of one particular sector on the output, employment, and income levels in the complex. The sector under consideration is the Household sector, which is of particular interest for a variety of reasons. The column entries in the flow table corresponding to the Household column represent expenditures by households for the output of each industry. Therefore, a consideration of only the Household column of the final demand sector leads to a determination of the effects of changes in consumer expenditures on the levels of output, employment, and income for each industry in the complex. This information is of particular interest to the market analyst. Not only are the results of this undertaking quite informative, but the data required for this type of analysis may be obtained at a minimum cost.

Obtaining information regarding consumer expenditure patterns within a given complex is quite costly; in fact, such an undertaking may involve a larger amount of resources than did the collection of data for the construction of the input-output table. Fortunately, two

excellent studies have been made in this direction, the first being the Wharton Study in 1950¹. Although the results of this study are thirteen years old, they are informative and may be used in the present model. However, the use of these data would involve the assumption of constant consumer expenditure patterns which is far more difficult to justify than that of constant activity coefficients for industries within the complex. The Bureau of Labor Statistics has recently completed a study titled, A Survey of Consumer Expenditures² for the year 1960-61. This study updates the Wharton Study and can be used as a guide in determining the expenditure patterns of consumers in a given area. Since the final results of this study are not published, they could not be used in determining household expenditures in Duval County. The values for household expenditures which are listed in Table 5.2-6 are estimates based on the percentage of each industry's output which went to the household sector for the year 1947. This assumption, which implies that these percentages apply to Duval County, was necessary in order to give a numerical example of the methodology to be used in determining the contribution of consumer expenditures to the aggregate levels of output, employment, and income. The results of this analysis are summarized in Table 5.2-6.

¹Wharton School of Finance and Commerce, Study of Consumer Expenditures, Incomes, and Savings. Vol. XVIII, Summary of Family Incomes, Expenditures, and Savings All Urban Areas Combined - 1950: U. S. Department of Labor, Bureau of Labor Statistics, 1957.

²U. S. Department of Labor, Bureau of Labor Statistics. Survey of Consumer Expenditures 1960-1961. Advance Report of Consumer Expenditures and Income: 1963,

Table 5.2-6 shows that for consumer food expenditures totaling \$48,733,162, the Food industry must produce an output of \$57,124,535. Likewise for a consumer demand equal to \$23,407, the Fabricated Metal industry must produce an output equal to \$40,048. The column of outputs was obtained by multiplying the column of household expenditures by the matrix formed by the interdependence coefficients. Conceptually, household expenditures was treated as the only element in the final demand sector which is analogous to converting final demand to its corresponding output. By using this modification of the model the required output, for a given level of consumer demand, of any industry in the complex can be determined. Related to these outputs are the corresponding levels of employment and income which were obtained by the methods in Chapter 3. This modification is of particular interest in determining the effects of changes in consumer demand on income received within the given complex.

TABLE 5.2-6--ESTIMATED LEVELS OF CONSUMER EXPENDITURES AND THEIR CORRESPONDING OUTPUT BY INDUSTRY TYPE IN DUVAL COUNTY

Industry	Household Expenditures (Dollars)	Output (Dollars)	Employment	Income ((\$1,000))
Food	48,733,162	57,124,535	1,599	6,455
Lumber	72,446	1,297,877	139	425
Furniture	3,991,370	4,026,770	345	1,363
Paper	339,817	2,916,183	129	765
Printing	3,158,885	3,680,104	368	1,941
Chemical	2,876,504	6,775,073	238	1,085
Stone, Clay, and Glass	1,025,804	2,586,526	185	736
Fabricated Metal	23,507	40,048	11	54
Transportation	9,073,164	13,130,102	602	3,044
Eating and Drinking	1,276,913	1,331,851	127	223

Source: W. D. Evans, M. Hoffenberg, "The Interindustry Relations Study for 1947, The Review of Economics and Statistics, XXXIV, (1952), 97-142.

The Multiplier and Its Effects on Output,
Employment, and Wages

The previous extensions of the basic model discussed the effects of a given schedule of final demand or consumer expenditures on output, income, and employment levels. The analysis was not concerned with relating a change in final demand to a change in output. By using the output multiplier the change in final demand for the products of an industry can be related to its corresponding change in output, employment, and income.

Output Multiplier.-- Although the concept of the output multiplier is cited in the current literature, it is called various names. Since the California Study³ has made an extensive use of the output multiplier, this thesis shall also refer to it as the output multiplier. As previously stated, activity coefficients of the form a_{ij} state the direct output of industry i for each dollar of output by industry j . The interdependence coefficients A_{ij} state the direct and indirect output from industry i required for each dollar of final demand produced by industry j .

An important difference between activity coefficients and interdependence coefficients is that the former apply to output whereas the latter apply to final demand. Interdependence coefficients are discussed in Chapters 2 and 4. Consider the following matrix of interdependence coefficients:

³W. E. Martin and H. O. Carter, A California Interindustry Analysis Emphasizing Agriculture, Giannini Foundation Report No. 250 (Berkeley, Calif.: California Agriculture Experiment Station, University of California, February, 1962).

$$\begin{bmatrix}
 A_{11} & A_{12} & A_{13} & . & . & A_{1n} \\
 A_{21} & A_{22} & A_{23} & . & . & A_{2n} \\
 A_{31} & A_{32} & A_{33} & . & . & A_{3n} \\
 . & . & . & . & . & . \\
 . & . & . & . & . & . \\
 A_{n1} & A_{n2} & A_{n3} & . & . & A_{nn}
 \end{bmatrix}$$

By examining the elements in any column of the matrix the following conclusions are made: Considering column 3, A_{13} shows the direct and indirect output from industry 1 so that industry 3 may deliver one dollar of final demand; A_{23} shows the output required from industry 2 so that industry 3 may deliver one dollar of final demand. Therefore, A_{ij} represents the output required from industry i so that industry j may deliver one dollar of final demand. Each column total will be designated as an output multiplier since it states the output required for each dollar of final demand. The total of the sums of each column, i.e., $\sum A_{ij}$ is the total output which must be produced by the entire complex in order that one dollar of final demand be produced by industry j. For any industry, the multiplier represents the change in gross output in Duval County resulting from an increase of one dollar in the final demand for the output of that industry. Table 5.2-7 shows that an increase of one dollar in the final demand for the products of the Food industry will increase its output in Duval County by 1.27489. Similarly for every dollar increase in the final demand of the Lumber industry the gross output within the complex will increase by 1.24264.

TABLE 5.2-7--OUTPUT MULTIPLIERS FOR THE BASIC MODEL

Industry	Multiplier	Rank
Food	1.27489	6
Lumber	1.24264	7
Furniture	1.22386	8
Paper	1.65393	1
Printing	1.47570	3
Chemical	1.35598	5
Stone, Clay, and Glass	1.20657	9
Fabricated Metal	1.03845	10
Transportation	1.49982	2
Eating and Drinking	1.41080	4

Source: Table 4.3-1.

The applications of the input multiplier to marketing analysis are presented in the next chapter. A multiplier which is closely related to the output multiplier is the weighted output multiplier. The weighted output multiplier is defined in the following manner:⁴

$$(5.2-1) \quad \sum \Delta X_i = \sum A_{ij}(\Delta Y_j)$$

Where $\sum X_i$ is equal to the total change in output for the n industries in the complex associated with a specific final demand change (ΔY_j) for sector j , $\sum A_{ij}$ is the output multiplier for sector j , and ΔY_j is its corresponding weight.

The change in output resulting from a 10 percent change in final demand is obtained by applying equation (5.2-1) to each final demand in Table 5.2-1. From Table 5.2-8, a 10 percent change in the demand for

⁴Ibid. p. 13.

TABLE 5.2-8--CHANGE IN OUTPUT INDUCED BY A TEN PERCENT CHANGE IN FINAL DEMAND

Industry	(1) Final Demand ^a (Dollars)	(2) Change in Output ^b (Dollars)	(3) Percentage Change in Output ^c	(4) Rank of Weighted Multiplier ^d	(5) Rank of Unweighted Multiplier ^e
Food	128,028,516	16,322,227	10.8	1	6
Lumber	4,957,523	616,041	5.9	9	7
Furniture	7,784,528	952,717	12.0	8	8
Paper	22,690,764	3,752,893	8.7	3	1
Printing	13,516,330	1,994,604	12.8	5	3
Chemical	16,860,128	2,286,199	7.4	4	5
Stone, Clay and Glass	12,902,519	1,556,779	9.1	7	9
Fabricated Metal	4,073,111	422,976	10.1	10	10
Transportation	41,387,522	6,207,383	10.4	2	2
Eating and Drinking	13,421,030	1,893,425	13.5	6	4

^aTable 4.2-1^bChange in output = (Final Demand)(Change in Final Demand)(Output Multiplier,
Table 5.2-7)Example: $16,322,227 = (128,028,516)(.10)(1.27489)$ ^cPercentage change in output = $\frac{\text{Change in output}}{\text{Original output}}$ Example: $10.8 = \frac{16,322,227(100)}{149,855,762(\text{Table 5.2-1})}$ ^dRanking of the data in column (2), from highest to lowest^eTable 5.2-7

the products of the Food industry increases gross output in the area by \$16,322,227, which represents an increase of 10.8 percent in output of the Food industry. In column (3) the percentage changes in output are very close to the 10 percent change in final demand; the reason for this is that in the majority of industries the final demand sector accounts for a very large share of the total output of the industry. Such an occurrence would be eliminated if there had been a larger number of industries in the endogenous sector thereby decreasing the magnitude of the final demand. Column (3) shows that a 10 percent increase in the final demand for the products of the Eating and Drinking industry results in a 13.5 percent increase in the output of that industry and an additional \$1,893,425 of output for the entire area. Therefore, based on a percentage basis, an increase in the final demand for the Eating and Drinking industry offers the greatest percentage increase in aggregate output. These percentage changes in industry output are quite impressive but the magnitude of the increase is often considered to be of greater significance since a greater output results in a larger increase in employment and income. For example, a 10 percent increase in final demand for the products of the Printing industry results in a 12.8 percent increase in output which is 0.7 percent smaller than the increase attributed to the Eating and Drinking industry. However, the absolute increase in output of the Printing industry exceeds that of the Eating and Drinking industry by approximately \$100,000.

Employment Multiplier.-- The preceding analysis discusses the methodology involved in converting a change in final demand to a change in output. Since employment and income levels are directly related to output levels, an examination of the role of the multiplier in determining the effects of a change in final demand on the levels of employment is interesting. Therefore, this section describes the methodology involved in relating a change in final demand to a change in employment.

In Chapter 3, employment levels are determined from a given final demand schedule. The objective of the present analysis is to examine the change in employment resulting from a change in final demand. The methodology involved in determining changes in employment from changes in final demand is directly analagous to converting changes in final demand to changes in output levels.

The output multiplier, which is defined by equation 5.2-1, is not directly applicable to both problems. For the estimation of employment levels, instead of output levels, the terms in the equation must be redefined. In addition to redefining the terms, the existing data must be modified in order that they be used in the new equation.

By definition let:

l_j = the amount of labor required by industry j in the production of an output X_j .

g_j = the activity coefficient for labor.

In the previous derivation of the activity coefficients, a linear relationship was assumed, i.e.,

$$x_{ij} = a_{ij}X_j$$

$$x_{ij} = f(X_j)$$

A Similar assumption will be made in the present analysis, i.e.,

$$l_j = \phi(X_j)$$

Since the present analysis is confined to labor there is no need for two subscripts.

Since l_j was defined as the activity coefficient for labor

$$(5.2-2) \quad l_j = g_j X_j$$

$$g_j = \frac{l_j}{X_j}$$

Remembering that
$$X_j = \sum_{i=1}^n x_{ij}$$

$$(5.2-3) \quad \text{Similarly } L = \sum_{j=1}^n l_j$$

Substituting equation (5.2-2) into (5.2-3)

$$(5.2-4) \quad L = \sum_{j=1}^n g_j X_j$$

Therefore equation (5.2-4) may be written in matrix form

$$(5.2-5) \quad L = (G)(X)$$

However

$$(5.2-6) \quad X = (I-A)^{-1}Y$$

Substituting (5.2-6) into (5.2-5)

$$(5.2-7) \quad L = G(I-A)^{-1}Y$$

Equation (5.2-7) is an interesting equation and should be examined in detail. Given a final demand schedule Y , the model (equation) will determine the level of employment necessary to produce the output

required by the demand schedule. This extension appears to be similar to the discussion concerning the conversion of output to employment in Chapter 3. However there is one basic difference, namely the conversion presented in Chapter 3 applied national productivity data to a particular region whereas the present extension uses the productivity figures obtained from a direct analysis of the given region. Undoubtedly, it is advantageous to use productivity values obtained from the region under consideration; however, on occasion this may be difficult since the total output of the industry may be unknown.

In its present form, equation (5.2-7) does not relate the effects of a change in the final demand for the products of an industry to a change in labor requirements. However it may be written in the following form:

$$(5.2-8) \quad \Delta L_j = (G)(A_j)(\Delta Y)$$

where A_j is the column vector in the interdependence matrix corresponding to industry j . The economic interpretation of the interdependence coefficient A_{ij} represents the direct and indirect output required from industry i so that industry j could furnish one dollar of final demand. Similarly $(G)(A_j)$, the employment multiplier, indicates the amount of labor required both directly and indirectly by industry j so that it may produce one dollar of final demand.

The results of applying equation (5.2-8) to the final demand data in Table 5.2-8 are listed in Table 5.2-9. Columns (1) and (2) contain employment and output data for Duval County. The elements in column (3) were obtained by computing the ratio of output to employment

TABLE 5.2-9--CHANGE IN EMPLOYMENT INDUCED BY A TEN PERCENT CHANGE IN FINAL DEMAND

Industry	(1) Employment ^a	(2) Output ^b (Dollars)	(3) Direct Employment per \$1000 of Output ^c	(4) Direct and Indirect Employment per \$1000 Final Demand ^d	(5) The Effect of a 10% Change in Final Demand on Employment	(6) Percentage Change in Total Employment Resulting from a 10% Change in Final Demand ^e
Food	4195	149,855,762	.027994	.0381354	488	2.55
Lumber	1107	10,269,362	.107796	.1227051	61	0.32
Furniture	682	7,943,124	.085860	.1068955	83	0.43
Paper	1090	43,002,534	.044392	.0726059	165	0.86
Printing	1560	15,573,448	.100170	.1285989	173	0.91
Chemicals	1093	30,988,309	.035271	.0527243	88	0.46
Stone, Clay, and Glass	1227	17,113,840	.071696	.0836226	107	0.56
Fabricated Metal	1144	4,202,438	.272222	.2782710	113	0.59
Transportation	2746	59,884,795	.045854	.0696969	288	1.51
Eating and Drinking	1341	14,036,380	.095537	.1139854	152	0.80

^aObtained from Table 4.1-1^bObtained from Table 4.2-1^cColumn (1) divided by column (2)^dObtained by multiplying the row vector of employment coefficients listed in column (3) by the column vector of interdependence coefficients (Table 4.3-1) corresponding to the sector indicated at the left. Example: $(.027994)(1.16173) + (.107796)(.00597) \dots = (.095537)(0.0) = .0381354$ ^eChange in employment = (Final Demand, \$1,000)(.10)(Column (4) coefficients)^fObtained by dividing each element in column (5) by 19091, which is the total number of employees in Duval County. Total employment in the above industries is 17,004 which represents 89 percent of the employees engaged in manufacturing industries in Duval County.

for each industry, and may be interpreted as activity coefficients for labor. However, activity coefficients determine only the direct labor requirements per \$1000 of output, i.e., there is no consideration given to the indirect requirements for labor.

In order to obtain both the direct and indirect labor requirements per \$1000 of final demand, the term $GA_j, j=1,2,\dots,n$, must be computed by multiplying the row vector in column (3) by each column vector in the matrix formed by the interdependence coefficients. The product of this answer and the change in final demand determines the effect of the change in final demand on employment.

From Table 5.2-9, a 10 percent increase in final demand increases employment in the Food industry by 488. Likewise employment in the Lumber, Furniture, and Paper industries increased by 65, 83, and 165, respectively.

Column (6) gives the percentage change in total employment resulting from a 10 percent increase in final demand. The usefulness of the model in studying structural changes in the complex is beyond comprehension. For example, in the Furniture and Chemical industries, a 10 percent increase in final demand has nearly the same effect on employment in both of these industries, 83 additional workers in the Furniture industry and 88 in the Chemical industry. Now examine the dollar changes in final demand necessary to generate this additional employment. From Table 5.2-8, a 10 percent change in final demand is equal to \$778,453 for the Furniture industry and \$1,686,013 for the Chemical industry. Therefore, for every dollar of final demand expenditures received by the Furniture industry, approximately two dollars

must be received by the Chemical industry in order that the change in employment be equal for both industries.

The Food industry has the greatest effect on employment since a 10 percent increase in final demand generates 488 additional jobs. However, this statistic must be interpreted in conjunction with the employment multiplier which is given in column (4), Table 5.2-9. The employment multiplier for the Food industry is 0.38 which means that for every increase of \$1,000,000 of final demand in the Food sector, 38 additional jobs will be created. The same increase in final demand will add 123 employees to the Lumber industry payroll. Although the Food industry has the largest effect on employment, it also requires the greatest amount of expenditures.

Income Multiplier.-- The previous sections contain a discussion of the output and employment multipliers. This section concentrates on the income multiplier. The derivation of the income multiplier is analogous to that of the employment multiplier. In fact, equation (5.2-8) can be written in the more general form:

$$(5.2-9) \quad \Delta Q_j = Q_a A_j (\Delta Y) \quad (j = 1, 2 \dots n)$$

Where:

ΔQ_j = the change in any resource Q required by industry j so that a given schedule of final demand may be delivered by j .

Q_a = row vector of activity coefficients for resource Q .

A_j = column vector in the interdependence matrix corresponding to industry j .

ΔY = The change (dollars) in final demand.

Table 5.2-10 was obtained by considering income as the resource Q and applying equation (5.2-9) to the income data. Column (3) represents the direct income received by the employees per \$1000 of output. Income in the present context is defined as the gross earnings paid to the employees of each industry. All forms of compensation are included in this figure such as salaries, wages, and commissions. Excluded from this income figure are payments to the proprietor or partners of unincorporated businesses. Direct and indirect income per \$1,000 of final demand is calculated by computing $Q_e A_j$. The effect of a 10 percent change in final demand on income is calculated from equation (5.2-9).

The income multipliers in column (4) show that a change in final demand in the Printing industry has the greatest effect on income levels since \$684.83, the largest amount, is generated by a 10 percent increase in final demand. This represents a 1.2 percent increase in income for the entire complex. The Food industry has the lowest income multiplier in the entire complex. A 10 percent change in the final demand of the Food industry necessitates a \$13 million expenditure whereas a comparable change in the Lumber industry requires a \$.5 million change in expenditures. Column (6) shows that the effect of the Food industry on income is 10 times that of the Lumber industry, but that the expenditures required by the Food industry are 24 times as large as those required by the Lumber industry.

The Furniture and Eating and Drinking industries have identical effects on aggregate income. However, expenditures of only \$778,453 are required for the Furniture industry while \$1,342,103 are required for the Eating and Drinking industry.

TABLE 5.2-10--CHANGE IN INCOME INDUCED BY A TEN PERCENT CHANGE IN FINAL DEMAND

Industry	(1) Payroll ^a (\$1000)	(2) Output ^b (Dollars)	(3) Direct Income per \$1000 Output ^c	(4) Direct and Indirect Income for \$1000 of Final Demand ^e	(5) Change in Final Demand on Income ^f	(6) Percentage Change of Total Income Resulting from a 10% Change in Final Demand ^g
Food	16957	149,855,762	113.15	156.79	2,007	2.61
Lumber	3392	10,269,362	330.30	407.74	202	.26
Furniture	2695	7,943,124	339.29	407.04	317	.41
Paper	11332	43,002,534	263.52	419.77	952	1.24
Printing	8234	15,573,448	528.72	684.83	925	1.20
Chemical	4983	30,988,309	160.80	233.44	394	.51
Stone, Clay, and Glass	4941	17,113,840	288.71	341.99	441	.57
Fabricated Metal	(d)	4,203,438	450.00	462.80	189	.25
Transportation	13888	59,884,795	231.91	349.17	1,445	1.88
Eating and Drinking	2353	14,036,380	167.64	238.18	320	.41

^aObtained from Table 4.4-2.

^bObtained from Table 4.2-1.

^cObtained by dividing each element in column (1) by its corresponding entry in column (2).
^dThe direct wages per \$1000 of output were not calculated directly from the Duval Data since the income per worker in Duval County is greater than the national average productivity of workers in this industry (Table 4.4-1). Therefore if this data were used in the calculation of direct wages per \$1000 of output, the value of the wages would exceed the value of the output of that industry. This condition will not exist when the analysis is based on the data obtained from a survey. \$450 represents an average value based on other interindustry studies.

^eObtained by multiplying the row vector of income coefficients listed in column (3) by the column vector of interdependence coefficients (Table 4.3-1) corresponding to the sector indicated at the left. Example: $(113.15)(1.16174) + (330.30)(.00597) \dots (167.64)(0.0) = 156.79$

^fChange in wages = (Final Demand)(.10)(Column (4) coefficients)

^gObtained by dividing each element in column (6) by \$76,863,000, the total payroll in Duval County. The above industries account for 90 percent of this total.

An Aggregate Model Constructed From the Basic Model

Previous discussions have considered the advantages in performing a pilot study prior to conducting a survey. The results of the pilot study furnish data which permit a more intelligent design of the interindustry model. After the pilot study is completed, the staff of interviewers begin the survey. The survey furnishes the interindustry group with reams of information to classify, aggregate, and analyze. The objective of the present modification is to examine the methodology involved in conducting a preliminary analysis of the data obtained from the survey.

There are several advantages in performing a preliminary study before beginning the detailed analysis. First, the task of analyzing all the data obtained from the survey is very great. The resources which are allocated for the study may be insufficient for its completion. Inaccurate estimates of the resources required for an interindustry study are not unusual since it is difficult to attach a direct dollar value to the cost of contacting each different datum source. In the event that there are not enough resources available to complete the detailed analysis, the modification described in this section provides a method for obtaining some information from the detailed data.

The second advantage in performing this preliminary analysis is that it may decrease the time required for the completion of the study. Possibly there will be situations in which an interindustry model is used to solve a short-run problem and hence there may be no time for a detailed analysis of the data. This extension provides

a method for performing such an analysis. The modification under consideration condenses data in the form of a m by n matrix into one of $(m-k)$ by $(n-k)$.

In Table 4.2-1, the flows in the exogenous sector are not distributed over each component in the final demand sector, i.e., the final demand figure is one aggregate figure. Table 5.2-11 contains a complete breakdown of interindustry flows for the Duval complex. The interindustry flows in the endogenous sector are identical to those in Table 4.2-1. The difference between Table 4.2-1 and Table 5.2-11 is that the latter distributes the aggregate final demand figure over its component parts whereas the former reports one total figure for final demand. Table 5.2-12 contains the activity coefficients for the data in Table 5.2-11. Although the activity coefficients for the exogenous sector are not considered constant, they were calculated for use in other modifications of the model.

As described in Chapter 3, the methodology involved in allocating the final demand over its component parts is quite arbitrary. Average ratios from other interindustry studies were examined and then applied to the current model. Therefore, the entries in the exogenous sector may or may not resemble the actual conditions in Duval County. However, neither the activity coefficients nor the interdependence coefficients are dependent on exogenous flows since they are derived from endogenous flows.

The basic and most important step in this modification is the aggregation of industries. The aggregation of industries is a

TABLE 5.2-11--INTERINDUSTRY FLOWS
(Dollars)

Industry	Food	Lumber	Furniture
1. Food	19,762,006	585	993
2. Lumber	325,799	1,871,701	1,063,440
3. Furniture	---	---	19,328
4. Paper	1,823,044	7,786	41,725
5. Printing	157,652	---	---
6. Chemical	5,841,305	44,768	172,993
7. Stone, Clay, and Glass	1,016,628	24,628	94,562
8. Fabricated Metals	---	---	13,679
9. Transportation	10,040	1,602	---
10. Eating and Drinking	---	---	---
11. Construction	464,862	19,511	19,063
12. Government	4,513,668	578,165	307,398
13. Inventory Change	1,589,531	---	38,126
14. Gross Private Capital Formation	DEPRECIATION AND OTHER CAPITAL CONSUMP-		
15. Households	24,837,643	4,386,044	2,919,098
16. Imports	89,513,584	3,334,572	3,252,719
Total	149,855,762	10,269,362	7,943,124

Industry	Transpor- tation	Eating and Drinking	Construc- tion
1. Food	---	366,906	---
2. Lumber	195,404	506	2,509,725
3. Furniture	108,092	---	541,404
4. Paper	138,393	5,980	380,524
5. Printing	---	3,102	---
6. Chemical	465,544	4,486	928,700
7. Stone, Clay and Glass	808,025	6,190	5,231,018
8. Fabricated Metals	40,601	---	---
9. Transportation	18,482,962	1,470	104,260
10. Eating and Drinking	---	566,920	---
11. Construction	179,654	77,200	---
12. Government	2,748,709	1,490,663	1,000,000
13. Inventory Change	23,953	---	2,000,000
14. Households	13,863,330	4,498,659	1,000,000
15. Imports	22,830,128	7,014,298	---
Total	59,884,795	14,036,380	13,695,631

Source: Table 4.2-1.

TABLE 5.2-11--(Continued)

Paper	Printing	Chemical	Stone Clay Glass	Fabricated Metal
163,581	825	1,525,368	6,982	---
1,683,893	1,463	100,157	60,210	9,266
31,176	---	---	---	---
14,294,214	2,620,336	737,459	633,263	9,220
---	1,861,286	35,078	---	---
1,008,581	234,738	5,915,575	408,274	31,917
156,572	---	574,647	1,519,110	10,959
---	---	---	---	75,047
---	---	---	2,669	---
---	48,430	---	---	---
227,913	35,818	77,470	118,085	8,825
2,704,859	808,261	1,679,566	1,139,781	204,658
473,027	62,293	306,784	---	---
TION ALLOWANCES ARE INCLUDED IN THE HOUSEHOLD ROW				
11,761,193	7,328,864	7,564,246	7,966,492	1,710,812
10,497,525	2,571,134	12,471,959	5,258,974	2,141,734
43,002,534	15,573,448	30,988,309	17,113,840	4,202,438
Government	Inventory Change	Gross Private Capital Formation	Households	Exports
159,957	134,268	---	48,733,162	79,101,133
15,071	187,583	38,745	72,446	2,133,953
142,267	213,400	192,267	3,991,370	2,703,820
132,210	98,582	---	339,817	21,739,981
330,930	---	188,713	3,158,885	9,837,802
271,213	445,858	---	2,876,504	12,337,853
51,145	297,250	454,618	1,025,804	5,842,684
7,123	26,638	257,743	23,507	3,758,100
438,556	1,162,589	8,647,019	9,073,164	21,960,464
---	---	---	12,762,890	658,140
800,000	---	3,500,000	100,000	---
2,000,000	---	---	18,000,000	---
---	500,000	---	---	---
25,000,000	---	---	---	20,000,000
1,000,000	---	---	22,000,000	---
30,348,472	3,066,164	13,279,105	122,157,549	180,073,930

TABLE 5.2-12--ACTIVITY COEFFICIENTS

Industry	Food	Lumber	Furniture	Paper	Printing	Chemical	Stone Clay and Glass	Fabricated Metal
Food	.131870	.000057	.000125	.003804	.000053	.049224	.000408	.000000
Lumber	.002174	.182243	.133882	.039158	.000094	.003232	.003518	.002205
Furniture	.000000	.000000	.002433	.000725	.000000	.000000	.000000	.000000
Paper	.012165	.000758	.005253	.332409	.168256	.023798	.037024	.002194
Printing	.001052	.000000	.000000	.000000	.119509	.001132	.000000	.000000
Chemical	.038979	.004359	.021779	.023454	.015073	.190897	.023855	.007595
Stone, Clay, and Glass	.006784	.002398	.011905	.003641	.000000	.018544	.088765	.002608
Fabricated Metal	.000000	.000000	.001722	.000000	.000000	.000000	.000000	.017858
Transportation	.000067	.000156	.000000	.000000	.000000	.000000	.000156	.000000
Eating and Drinking	.000000	.000000	.000000	.000000	.000311	.000000	.000000	.000000
Construction	.003099	.001899	.002399	.005299	.002299	.002499	.006899	.020999
Government	.030099	.056299	.038699	.062899	.051899	.054199	.066599	.048700
Inventory Change	.010599	.000000	.004799	.010999	.003999	.098999	.000000	.000000
Gross Private Capital Formation	-	-	-	-	-	-	-	-
Households	.166299	.427099	.367499	.273499	.470599	.244099	.465499	.407100
Imports	.596933	.324710	.409501	.244114	.165097	.402473	.307293	.509641

Source: Table 5.2-11.

TABLE 5.2-12--(Continued)

	Transportation	Eating and Drinking	Construction	Government	Inventory Change	Gross Private Capital Formation	Households	Exports
Food	.000000	.261397	.000000	.005271	.043789	.000000	.398936	.439270
Lumber	.003263	.000361	.183250	.000497	.061178	.002918	.000593	.011850
Furniture	.001805	.000000	.039531	.004688	.069598	.014479	.032673	.015015
Paper	.002311	.004261	.027784	.004356	.032152	.000000	.002781	.120728
Printing	.000000	.002210	.000000	.010904	.000000	.014211	.025859	.054632
Chemical	.007774	.003196	.067809	.008937	.145412	.000000	.023547	.068515
Stone, Clay, and Glass	.013493	.004410	.381947	.001685	.096945	.034236	.008397	.032446
Fabricated Metal	.000678	.000000	.000000	.000235	.008688	.019410	.000192	.020870
Transportation	.308642	.000105	.007612	.014451	.379167	.651175	.074274	.121952
Eating and Drinking	.000000	.040390	.000000	.000000	.000000	.000000	.104478	.003655
Construction	.062999	.005499	.000000	.026361	.000000	.263572	.000818	.000000
Government	.045899	.106199	.073015	.065901	.000000	.000000	.147350	.000000
Inventory Change	.000399	.000000	.146031	.000000	.163070	.000000	.000000	.000000
Gross Private Capital Formation	-	-	-	-	-	-	-	-
Households	.231499	.320499	.073015	.823765	.000000	.000000	.000000	.111064
Imports	.381234	.499722	.000000	.032951	.000000	.000000	.180095	.000000

complex problem and is discussed at length in a later chapter. Many problems are encountered in aggregating the industries in the illustrative model since it has only ten industries in the endogenous sector. If a survey were performed each industry would have been broken down into its primary components. For example, instead of listing the Food industry as one entry there might be eight entries, such as Meat Products, Dairy Products, Canning and Preserving Fruits, Grain Mill Products, Bakery Products, Sugar, Confectionery and Related Products, and the Beverage industry. In this extension of the model, these sub-industry classifications would then be combined into one, the Food industry. However, for illustrative purposes the aggregation of industries may continue as illustrated in Table 5.2-13.

The Food industry, which is called Agriculture, was not combined with any other industry since there was no industry closely related to it. The Lumber, Furniture, and Paper industries were combined and titled Forestry Products. The title given to this aggregate industry is arbitrary. The industries were combined by a vertical addition of row entries in Table 5.2-11. For example, the output of Forest Products to the Agriculture Sector is \$2,148,843 and is equal to the sum of \$325,799 and \$1,823,044. Likewise, the gross output of the Forest Product sector is equal to \$61,215,020 which is the sum of \$10,269,362, \$7,943,124, and \$43,002,534.

Similar combinations are made in the remaining sectors; Printing and Chemicals are called Manufacturing; Stone, Clay, and Glass and Fabricated Metals are Building Supplies; Transportation and Eating

are Services, Government, Construction and Inventory Change are Government. The Household and Import sectors are not combined with any other industry. As previously stated, the above industry combinations would be different if the model had more industries. The purpose in aggregating the industries is to provide a numerical example of this modification to the basic model.

The distribution coefficients in Table 5.2-14 are obtained from the interindustry flows in Table 5.2-11. The distribution coefficients are defined by the following equation $d_{ij} = \frac{x_{ij}}{X_i}$, i.e., each element in the table was divided by its row total. The distribution coefficients show the manner in which the output of an industry is distributed to the other industries in the complex. Hence 3.5 percent (.035103 out of every dollar of output), of the Forestry industry output goes to Agriculture, 31 percent remains in the Forestry industry and 7.5 percent goes to Household. A similar analysis may be applied to each industry in the model.

Table 5.2-15 contains the coefficients for the transactions table (Table 5.2-11). Table 5.2-16, the matrix of interdependence coefficients, is obtained from the activity coefficients by the methods stated in Chapter 2. The economic interpretations of the activity and interdependence coefficients are identical to those described in the previous modifications. The only difference between the present coefficients and those used in the previous extensions is that the former apply to the aggregate industries instead of the individual industry listed in the original model.

TABLE 5.2-13--AGGREGATED INTERINDUSTRY FLOWS
(Dollars)

Industry	Agriculture	Forestry	Manufacturing	Building Supplies
Agriculture	19,762,006	165,159	1,526,193	6,982
Forestry	2,148,843	19,013,263	3,459,415	711,959
Manufacturing	5,998,957	1,226,342	8,046,677	440,191
Building Supplies	1,016,628	289,441	574,647	1,605,116
Services	10,040	1,602	48,430	2,669
Government	6,468,061	4,368,062	2,970,192	1,471,349
Households	24,937,643	19,066,335	14,893,110	9,677,304
Imports	89,513,584	17,084,816	15,043,093	7,400,708
TOTAL	149,855,762	61,215,020	46,561,757	21,316,278

Source: Table 4.2-1.

TABLE 5.2-13 --(Continued)

Industry	Services	Government	Households	Exports	Total
Agriculture	366,906	294,221	48,733,162	79,101,133	149,855,762
Forestry	448,375	4,220,766	4,634,645	26,577,754	61,215,020
Manufacturing	473,132	1,976,701	6,224,102	22,175,655	46,561,757
Building Supplies	854,816	5,613,174	1,761,672	9,600,784	21,316,278
Services	19,051,352	1,705,405	30,483,073	22,618,604	73,921,175
Government	4,520,179	6,300,000	21,600,000	---	47,797,843
Households	18,361,989	26,000,000	---	20,000,000	132,936,381
Imports	29,844,426	1,000,000	22,000,000	---	181,866,627
TOTAL	73,921,175	47,110,267	135,436,654	180,073,930	715,490,843

TABLE 5.2-14--DISTRIBUTION COEFFICIENTS

Industry	Agricul- ture	Forestry	Manufac- turing	Building Supplies	Services	Govern- ment	House- holds	Exports
Agriculture	.131786	.001101	.010178	.000047	.002447	.001962	.324980	.527496
Forestry	.035103	.310598	.056513	.011630	.007325	.068950	.075711	.434170
Manufacturing	.128839	.026338	.172817	.009454	.010161	.042453	.133674	.476263
Building Supplies	.047693	.013578	.026958	.075300	.040102	.263328	.082644	.450397
Services	.000136	.000022	.000655	.000037	.547725	.023071	.412373	.305983
Government	.137413	.091386	.062141	.030783	.094569	.131805	.451903	.---
Households	.187590	.143425	.112032	.072797	.138127	.195582	.---	.150448
Imports	.492139	.093931	.082706	.040689	.164083	.005498	.120954	.---

Source: Table 5.2-13.

TABLE 5.2-15--ACTIVITY COEFFICIENTS

Industry	Agricul- ture	Forestry	Manufac- turing	Building Supplies	Services	Govern- ment	House- holds	Exports
Agriculture	.131786	.002698	.032778	.000328	.004963	.006245	.359823	.439270
Forestry	.014330	.310598	.074297	.033400	.006066	.089593	.034220	.147594
Manufacturing	.040004	.020033	.172817	.020650	.006400	.041959	.045956	.123148
Building Supplies	.006780	.004727	.012342	.075300	.011564	.119150	.013007	.053316
Services	.000067	.000026	.001040	.000125	.257725	.036200	.225073	.125145
Government	.043800	.071356	.063790	.069025	.061149	.133729	.159484	---
Households	.166300	.311465	.319857	.453987	.248400	.551897	---	.111066
Exports	.596933	.279095	.323078	.347186	.403733	.021227	.162438	---

Source: Table 5.2-13

TABLE 5.2-16--INTERDEPENDENCE COEFFICIENTS

Industry	Agriculture	Forestry	Manufacturing	Building Supplies	Services
Agriculture	1.154093	.008782	.046553	.001766	.005523
Forestry	.030554	1.454891	.132734	.055526	.014101
Manufacturing	.056294	.012144	1.212698	.027544	.033194
Building Supplies	.007034	.018401	.018095	1.082504	.010524
Services	.000185	.000103	.001714	.006224	1.347232

Source: Table 5.2-15

The Import Problem

The objective of the import problem is to examine the effects of output and final demand levels on the imports in the complex. The magnitude of the import flows are of particular interest since they represent the relationship between the given complex and the remainder of the world. The present analysis will not answer any questions regarding the advantages or disadvantages of imports into a region; however it will present a method of obtaining the relevant data upon which the analysis may be based.

The activity coefficients which apply to the import sector are expressed in terms of direct imports per dollar of output. In order to obtain this information, a complete breakdown of interindustry flows is necessary both in the endogenous and the exogenous sectors of the flow table. Therefore, the import activity coefficients are obtained from Table 5.2-12. In essence these represent direct imports per dollar of output and are calculated by dividing each entry in the Import row by its corresponding column total. The results of these calculations are listed in Table 5.2-17. Therefore for every dollar of output by the Food industry, sixty cents (.59693) worth of foreign goods must be imported. Likewise twenty-four cents (.2441) must be imported by the Paper industry.

A knowledge of direct imports per dollar of output is helpful but it does not include a complete analysis of the import problem. As previously discussed, both the direct and indirect requirements should be considered. Again the concept of the multiplier solves this problem; in this case it is called the import multiplier.

TABLE 5.2-17--IMPORTS GENERATED BY THE OUTPUT OF THE ENDOGENOUS INDUSTRIES

Industries	Direct Imports Per Dollar of Output	Direct and Indirect Imports per Dollar of Final Demand	Effects of 10% Change in Final Demand on Imports (\$1000)
	(1)	(2)	(3)
Food	.59693	.73202	9371
Lumber	.32471	.40530	200
Furniture	.40950	.48431	377
Paper	.24411	.44241	1004
Printing	.16510	.28152	381
Chemicals	.40247	.53378	900
Stone, Clay and Glass	.30729	.37151	479
Fabricated Metal	.50964	.56146	229
Transportation	.38123	.56934	2356
Eating and Drinking	.49972	.73108	981

(1) From row 17 Table 5.2-12

(2) Obtained by multiplying the row vector of import coefficients listed in column (1) by the column vector of interdependence coefficients (Table 4.3-1) corresponding to the sector indicated at the left.

Example: $(.59693)(1.16173) + (.32471)(.00597) + (.40950)(.00002)$
 $\dots\dots(.499722)(.0.0) = .73202$

(3) Change in imports = (Final Demand, Table(5.2-8))(.10)
 (Coefficients in column (2))

Example: $(128,028,516)(.10)(.73202) = 9371$

In the discussion of the income multiplier, equation (5.2-9) was stated to be general and could be applied to different types of resources. In this analysis, it will be used to determine the change in imports resulting from a change in final demand. The multiplier considers both the direct and indirect effects of imports due to a change in final demand. In order to obtain the import multiplier for any industry the row vector of activity coefficients is multiplied by the appropriate column of the interdependence matrix. The results of this multiplication are found in column (2) of Table 5.2-17.

From column (2), it is observed that for every dollar increase in the final demand for the products of the Food industry seventy-three (.73202) cents worth of import will enter the complex. Given a 10 percent change in final demand, \$9,371,000 worth of imports will enter the complex in the form of imports to the Food industry. Similarly, for every dollar increase in the final demand of the Chemical industry, \$900,000 worth of imports will enter the complex.

The Extension of the Basic Model to the Wholesale, Retail, and Service Sectors

Thus far, all of the industries in the endogenous sector, with the exception of the Eating and Drinking industry, are basically manufacturing industries. However, input-output analysis need not be confined to this sector, although the majority of the current interindustry models emphasize the manufacturing sector. The primary reason for this action is that the national interindustry study did not publish activity coefficients for the other sectors. Since the majority

of the regional studies base their analyses on the national coefficients, their analyses are also limited to the manufacturing sector.

However, in the event a survey is taken, the data would be available so that the wholesale, retail, and service sectors may be included in the model. The addition of these sectors poses no conceptual problems since the original derivation of the model is general and independent of the type of product produced by the industry. However, some of the assumptions made in interindustry analysis must be re-examined. The most important one is the assumption of constant activity coefficients. A good argument in favor of this assumption can be presented for the manufacturing sector. For example, it is quite possible that the amount of chemicals in the form of preservatives, used by the Lumber industry, is a fixed percentage of its output. The argument may also be extended to the service industries. For example, commercial laundries may use a fixed percentage of cleaning agents per unit of finished product.

However, when this assumption is applied to the wholesale and retail sectors, it may be more difficult to substantiate. For example, the amount of chemicals used by food stores in the retail sector for cleaning and related purposes may be independent of output. An empirical study should be undertaken before any generalization can be made on the validity of the assumption of constant activity coefficients in each of the sectors. The objective of this extension is not to undertake such a study, but to state that interindustry analysis can in theory be applied to many sectors of the economy.

The addition of these other industries to the model will affect both the exogenous and endogenous sectors of the model. In many inter-industry studies, the export column in the final demand sector is considered a residual, i.e., the output of an industry which cannot be attributed to any industry in either the endogenous or exogenous sector is counted as an export or an import. Therefore, with the addition of new industries in the endogenous sector, the value of exports will decrease and permit a more realistic estimate of the final demand figure.

The transactions matrix would be of the following form:

Industry Purchasing Industry Producing	Manufac- turing	Whole- sale	Retail	Service	Final Demand	Total Output
Manufacturing						
Wholesale						
Retail						
Service						
Final Demand						
Total Amount Input						

Each of the above sectors would contain the dominant industries in the complex corresponding to its particular sector. The transactions table could then be converted into an activity matrix which in turn could generate the interdependence coefficients. All of the previous modifications of the basic model could be made to the above transactions table.

CHAPTER 6

MARKETING INTERPRETATIONS OF THE BASIC MODEL AND ITS MODIFICATIONS

The introduction contains a discussion of the broadened domain of marketing thought. The analysis implies that the study of marketing involves not only an examination of the commodity, institutional, and functional approach, but also a complete examination of the structural interdependence of all firms in a given area. Alderson has called this synthesis functionalism. Functionalism, a recent innovation in marketing theory, is in an early stage of its development, and hence is subjected to a variety of criticisms.

One of the common criticisms of this approach is that it is too general and cannot be applied directly to current marketing problems. This section is a response to such a criticism. The application of Leontief input-output models to marketing implies an acceptance of functionalism. Basic to any interindustry model, is the structural relationship between seemingly autonomous sectors. These relationships form the foundations of functionalism.

As previously stated, functionalism, in addition to increasing the domain of marketing analysis, also consolidates the commodity, institutional and functional approach so as to provide a conceptual framework for decision making. The use of interindustry models for the

solution of marketing problems exemplifies this statement. The construction of the transactions matrix, which is a prerequisite for every interindustry model, requires large quantities of data. The commodity, functional, and institutional approaches to marketing not only aid in the collecting of data, but also provide a method for its classification and aggregation. The data must also be presented in a logical and consistent order. The above approaches to marketing are a means to this end.

Unfortunately, the knowledge obtained from any particular approach is of limited value unless it is combined with the results of the other approaches. The results of such a consolidation should then be subjected to many techniques of market analysis. However, the market analysis is usually terminated after using one of the three approaches.

There are several disadvantages to the traditional approaches to market analysis. First, the scope of the study is limited. This limitation may be intentional since there may be a limit on available resources. The other disadvantage and probably the most important one is that frequently there is no attempt to fuse the results of one approach with the results obtained from previous studies using a different type of methodology. This type of analysis is in complete disagreement with the basic concepts of functionalism.

At best, the results of the different approaches are usually combined but not used as effectively as possible. There are very few instances where a marketing principle, which was developed from any

of the preceding approaches, is related to other pertinent economic phenomena. A marketing principle, if it is to be a principle, cannot exist in a vacuum; it must be derived and applied to an existing situation in which none of the elements are independent.

Input-output analysis is one method of using the results of traditional marketing analysis to solve a variety of related problems. Undoubtedly, new and better methods will evolve; however, any successful method will be the one which emphasizes the effects of one firm on the marketing decisions of another. In the realm of possible use are the techniques of game theory and activity analysis. However, these techniques have not been useful in empirical studies as input-output analysis.

Unfortunately, the majority of the work done in the field of market research applies to the activities of an individual firm. It appears that the discipline is in need of general principles that may be applied in a variety of situations. One step in this direction is a consideration of the dependence of each firm on the remaining firms in the complex. Alderson has emphasized this need in his theory of functionalism; input-output models are a partial answer.

6.1 Marketing Uses of Interindustry Data

Each of the modifications of the basic model contained in Chapter 5 deals with a different facet of the interindustry model; however, the results obtained from each modification were transformed into regional estimates of output, employment, and income. The discussion

of each modification of the basic model contains an illustrative example of the manner in which the data generated by the extension may be used to solve a specific marketing problem. The remainder of this chapter describes the general uses of output, employment, and income data generated by the interindustry model without any reference to a particular modification of the basic model.

Usually the output estimates generated from an interindustry model are aggregate figures for all firms in each industry in the model. There are many uses of these aggregate figures. In periods of national mobilization, these output projections will state the output of any region for a given schedule of input. Conversely, the interindustry model will also determine if a particular region can produce the output required by the government. There are also many peace-time uses of the output levels generated by the model. In the past they have been used by local governments to study the effects of entering industries on employment and income levels within the given region.

The information generated from the model in the form of employment estimates is of extreme importance to the city planners. An advance knowledge of employment levels offers some information as to which industries are decreasing in their share of the output, and the qualifications of the employees who will soon become unemployed, thereby allowing the city to take remedial action. For example, a 10 percent increase in the final demand for Food products may result in only a 2 percent increase in employment, whereas a 10 percent increase in the final demand for Lumber products may result in an 8 percent increase in

employment. During periods of high unemployment this analysis will provide the state and local governments with information concerning those sectors of the economy they should patronize in order to obtain the greatest effect on increasing the aggregate level of employment in the community.

The regional planning board for a city would be interested in the outcome of the model since it could determine the effects of increased output on employment. With the increases in employment, new homes, roads, and schools would be needed near and around the location of the plants which are going to offer this additional employment.

There are many references to the uses of interindustry data by local governmental agencies and planning commissions.¹ However, there is nearly a complete absence in the literature of the uses of interindustry data for the individual firms in the region. Therefore, the current problem is to examine the method in which each firm in the region can use interindustry data in its market research activities.

The individual firm in the region may scale down the aggregate estimates of industry output by the use of scaling factors. One scaling factor may be obtained by assuming that the given firm's share

¹R. Dorfman, P. A. Samuelson, and R. M. Solow, Linear Programming and Economic Analysis (New York: McGraw-Hill, 1958); H. B. Chenery and P. G. Clark, Interindustry Economics (New York: John Wiley, 1959); W. Isard, Methods of Regional Analysis: An Introduction to Regional Science (New York: John Wiley, 1960); T. C. Koopmans, (ed.), Activity Analysis of Production and Allocation (New York: John Wiley, 1951); W. Leontief, Studies in the Structure of the American Economy (New York: Oxford University Press, 1953); O. Morgenstern, Economic Activity Analysis (New York: John Wiley, 1954).

of the market is constant. Given the increase in industry output, the individual firm could multiply this increase by an index representing its share of the increase in output. Therefore, if a 10 percent change in the final demand for the products of the Food industry results in a 2 percent change in output, the firms within the industry would be able to determine their individual change in sales. Having obtained its share of the increase in output, the firm may take appropriate actions in order to insure adequate warehousing and sales facilities.

A knowledge of this increase in output may affect the recruiting policies of the firm. For example, highly skilled workers may be reluctant to leave their present jobs and must be contacted well in advance of the time the given firm needs their services. Advance information generated by the model would be used by the personnel departments to forecast their labor requirements.

Similar questions could be answered concerning the necessary changes in a company's retraining program in order to move some of the existing semi-skilled workers into the skilled positions. The individual firm, knowing its relative share of the industry's employment, could calculate its labor requirements and guide its retraining programs accordingly.

A knowledge of employment data is also helpful to the individual firms in the business community. For instance, increased employment will indirectly affect the formulation of marketing programs. Increases in employment will lead to an increase in income which represents an increase in purchasing power for those employees in the various

industries. These changes in income levels give the individual merchants additional information on which to base their inventory policies, pricing policies, and sales promotion programs.

In essence, the interindustry model provides a means for evaluating market potentials. By the use of scaling factors the individual firm in the region can convert market potentials to sales potentials. The importance of interindustry models in market analysis is amplified when one remembers that sales potentials are made after estimating the market potentials for the given region. Hence, the interindustry approach to market analysis has a direct effect on the marketing decisions of the individual firm in the region.

CHAPTER 7

THE LIMITATIONS OF INTERINDUSTRY ANALYSIS

Input-output analysis provides solutions to many economic problems similar to those described in the previous chapters. However, a thorough examination of the limitations of the model should be made before rendering any decision based on the results of the model. The outcome of such an examination makes the analyst aware of the situations in which the results of the model represent accurate predictions of economic phenomena. Similarly it serves as a stern reminder of the disastrous effects which may result from an incorrect application of interindustry analysis.

The limitations of the analysis are a result of the assumptions made in the basic formulation of the model. These assumptions may lead to an unrealistic description of economic phenomena; if so, the model is only an approximate description of reality. The fact that the model is an approximation implies that there is an error in the results of the model, i.e., a difference exists between the results of the model and observable data in the economy. The objective of the current discussion is not to deny the presence of such an error, but to determine if it is significant.

To determine whether the errors are significant is by no means an objective undertaking. Presently, interindustry analysis does not make use of recent advances in mathematical statistics such as hypothesis

testing and decision-making. Furthermore none of the data are reported with their accompanying measure of reliability, such as their standard deviation. Such an obvious omission of this pertinent information is intentional. The most advanced statistical techniques cannot improve the basic theory underlying input-output analysis. Presently, many of the basic concepts of interindustry analysis are in the process of being modified in order to increase the scope and accuracy of the model. After the basic principles are formulated and agreed upon, advanced statistical techniques can then be used for a further refinement of the model. However, interindustry analysis is in its formative stage and therefore is not subjected to these advanced techniques. After the basic analytical structure of the model is formulated, the methods of statistical inference unquestionably lead to further improvements in the model. However, there are no great advantages in using advanced statistical techniques for testing hypotheses in an inaccurate or inconsistent model. Therefore, the empirical verifications of the model are made by comparing predictions with observed data.

7.1 A General Statement of the Assumptions

Chapter 2 contains a detailed statement of the assumptions made in the construction of the Leontief model. For the purpose of the present analysis, these assumptions may be stated in summary as:

1. One, and only one, technological process exists for the production of the homogeneous output of each industry.

2. The inputs to each industry are a function of only one independent variable, the output of that industry.

The assumptions underlying the construction of the model lead to three classes of errors; namely, the model error, statistical error, and computational error. The characteristics of each type of error are discussed in detail. The present analysis terminates with a review of the major studies in which the assumptions are subjected to empirical verification.

7.2 Model Error

Model error is the deviation of the results generated by the model from reality. Model error is inherent in every model since by definition a model only approximates reality. As the accuracy of the model increases, the model error decreases. The model error in the Leontief system may be traced to two specific causes; namely, fixed production coefficients and homogeneous industry output.

The other general categories of error; namely, computational error and statistical error, can be minimized to some extent by independent research in related disciplines. However, neither the most accurate computer nor the most accurate interindustry data can alone minimize the magnitude of the model error. Model error can be decreased only by an exhaustive examination and modification of the most basic theoretical concepts used in the construction of the model.

The Constancy of Activity Coefficients

Of all the assumptions made in the construction of the Leontief

model, the assumption of constant activity coefficients is the most controversial. It is the subject of many empirical tests using a variety of statistical techniques. Since the details of linear homogeneous production functions are discussed in Chapter 2, they are not repeated in this chapter. Although the production functions used in the model are represented by linear homogeneous functions, they need not be. Activity analysis, which involves the study of production processes (activities) instead of commodities, is a more general model of which the Leontief model is a special case. Activity analysis will handle nonlinear production functions. Actually the Leontief model could easily be modified in order to use linear nonhomogeneous functions. A further extension would be to deal with nonlinear functions.

Leontief recognized the restrictions of this assumption and agreed that the use of nonhomogeneous production functions might provide a better description of reality. However, little work has been done in the determination of production functions by industry type. Since there was no reason for selecting any other type of production function, Leontief used the most basic linear homogeneous function. The point to be emphasized is that although a basic assumption of the model is that the input to each sector is a function of its output, this by no means implies that the relationship of input to output must be represented by a linear homogeneous function.

Whether the production function is homogeneous, nonhomogeneous, linear or nonlinear, is not the primary issue of the production function

controversy. Most economists will agree on the importance of an expost calculation of the production coefficients for each industry. A consideration of the historical changes of these coefficients leads to important economic insights regarding the functioning of the economy. However, Leontief states that the activity coefficients are structural coefficients, i.e., they are fixed parameters which describe the economy at various intervals. In essence he states that they are invariant. Leontief does not mean to imply that the coefficients never change, but he does believe that the variation is not large enough to destroy the empirical validity of the model.

Calculation of the Activity Coefficients

The method by which the activity coefficients are calculated influences their accuracy through time. The most basic method of calculating the coefficients is described in Chapter 3. This method collects and summarizes all the relevant interindustry flows for the base-year and assumes that these ratios are valid for output predictions in future years. Experience shows that for certain sectors of the economy this assumption is valid, while for other sectors it is completely erroneous. Presently, research is being performed in order to refine the techniques involved in the calculation of the activity coefficients.

Direct Input Estimates.--One refinement centers around a historical study of the activity coefficients for different base-years. Such an analysis will give an insight into which coefficients vary

considerably with time. Therefore, a distinction can be made between those coefficients which change radically with time and those which remain relatively constant. Once this distinction is made, supplementary data sources may be used to obtain the input for those sectors whose activity coefficients change with time. The advantage in this approach is that the table can be kept up to date by reconstructing only a few sectors of the table instead of all sectors in the model.

Unfortunately, there are several disadvantages in this refinement. First, there is the problem of obtaining sufficient historical information in order to determine which activity coefficients change with time. Even if the tables are available for several years, there is no guarantee that those sectors whose activity coefficients show considerable change will continue to change. Similarly, those sectors whose activity coefficients remain relatively constant may begin to change. These occurrences are possible since no coefficient is entirely independent of the other coefficients in the model.

Second, direct input estimates represent a handicap to interindustry analysis if it is to be used for a predictive device. In order to derive the coefficients for those sectors whose activity coefficients show considerable change, projections of the inputs to each of these sectors must be made. Such input estimates will probably be related to pertinent economic variables. However, the projection is an approximation. If the projection is inaccurate, the activity coefficients derived from this approximation will be inaccurate.

Successive Linear Approximations.-- Using this method, the activity coefficients are represented by a curvilinear relationship. The production function is no longer represented by a straight line through the origin. Theoretically, it is represented by a curved line, but is approximated by a series of straight lines. Therefore, the activity coefficients are constant over different intervals of output. For example, between output of zero and X_1 , the activity coefficient is a_1 , between X_1 and X_2 , the activity coefficient is a_2 , etc.

From a theoretical standpoint, a curvilinear relationship is ideal. However, there is one great disadvantage in this approach, namely, the empirical derivation of the production function. In order to obtain the data necessary for the construction of this curvilinear relationship, the production process of each industry in the model must be studied in detail. With the present state of data collection procedures, the formulation of a curvilinear production function is somewhat impractical.

Two-Point Estimates: Nonhomogeneous Production Functions.-- The activity coefficients for the conventional Leontief model are usually obtained from a single set of observations, i.e., the transactions table is obtained for the base year from which the coefficients are calculated. Therefore the conventional activity coefficient represents a one-point estimate. As previously stated, one of the basic assumptions made in the construction of Leontief models is that the present one-point estimates of the activity coefficients are invariant.

This concept of invariance can be modified so as to state that the changes in the activity coefficients are constant. Therefore, the assumption that the one-point estimate is invariant is replaced by the assumption that the two-point estimate is constant. The conventional Leontief model is defined by the following equations:

$$(7.2-1) \quad \sum_j x_{ij} = X_i \quad (i, j, = 1, 2 \dots n)$$

$$(7.2-2) \quad x_{ij} = a_{ij}X_j$$

$$(7.2-3) \quad \sum_j a_{ij}X_j = X_i$$

Equations (7.2-1), (7.2-2), and (7.2-3) lead to one-point estimates; they are replaced by the following:

$$(7.2-4) \quad \sum_j (x_{ij}^1 - x_{ij}^0) = X_i^1 - X_i^0$$

$$(7.2-5) \quad x_{ij}^1 - x_{ij}^0 = a_{ij}(X_j^1 - X_j^0)$$

$$(7.2-6) \quad \sum_j a_{ij}(X_j^1 - X_j^0) = X_i^1 - X_i^0$$

The production function for the nonhomogeneous case is

$$(7.2-7) \quad X_{ij} = C_{ij} + A_{ij}X_j$$

Equation (7.2-7) shows that although the production function is linear, it is not represented by a straight line through the origin. Comparing equation (7.2-2) with equation (7.2-7) it is evident that the nonhomogeneous case has an additional parameter C_{ij} in addition to the

a_{ij} found in equation (7.2-1). Therefore, equation (7.2-7) must be solved from the data obtained from two input-output matrices. Starting with this system of equations, Professor A. Ghosh of the University of Cambridge has shown the following:¹

1. For any production function represented by

$$Y_r = a + bX_r + e_r$$

where: Y_r is an observed point in the input-output space, X_r is any random point in the output ordinate, and e_r is $N(0, \sigma^2)$, then the bias in the variance by the original Leontief function is

$$\sigma^2 \left(\frac{X_r}{X_1} \right)^2$$

2. if future outputs are projected for a period in which they are likely to differ from the earlier by approximately 70 percent, then the two-point estimates will generally lead to more accurate estimates of the activity coefficients.

The Problem of Aggregation

Another source of model error is due to the fact that all of the industries in the model are not identical single-product firms which is a direct violation of one of the basic assumptions. Aggregation in interindustry models is inevitable. The problem is not to decide between aggregation and non-aggregation but to decide upon the degree of aggregation.

¹A. Ghosh, "A Note on the Leontief Model with Non-Homogeneous Production Functions," Metroeconomica, XII (1950), 14-21.

Table 5.2-13 illustrates the aggregation problem. Before the industries were aggregated there were ten industries in the endogenous sector of the model, after the industries were aggregated there are only five industries in the endogenous sector of the model. The problem of aggregation is one of the most common problems encountered by the interindustry economist. It is impossible to construct a transaction matrix without aggregating industries. To date, the largest matrix constructed is a 500 x 500 table. Even this enormous table contains a great deal of aggregation since there is a multitude of products produced in the economy.

The problem of aggregation is of extreme importance since it influences the activity coefficients which in turn generate the interdependence coefficients. Changes in the interdependence coefficients lead to direct changes in the output projections based on a given final demand. The problem of aggregation is also complicated by the fact that one method of aggregation may be suited for aggregating industry i with industry j but may not be applicable to the aggregation of industry m with industry n.

The problem of aggregation is not entirely divorced from the previous problem of invariant activity coefficients; for this reason the aggregation problem is discussed in greater detail. For some industries the activity coefficients for a given product in process may remain constant but the activity coefficients in the transactions matrix may show a large change. This dichotomy is explained by a consideration of the product mix of the industry. With the passage of

time, a firm may increase its product line which increases the heterogeneity in its output figures.

Many of the aggregation problems in interindustry analysis are the result of a difference between definitions used in input-output models and those used in an industrial census. For example, secondary products are subtracted from each industry which manufactures one primary product and several secondary products, and then added to the industry for which the output is primary. Conceptually this is an easy undertaking; however, in practice it is a difficult task to perform. The difficulties center around the fact that the establishment is the basic unit of statistical data collection. Therefore, the census figures show an aggregate output for each industry without any regard to product.

The aggregation problem is not discussed in connection with the theoretical models of general equilibrium such as the Walrasian model. Conceptually, there is no difficulty in analyzing the simultaneous effects of a large number of variables. However, in an empirical application of the model, the millions of interindustry transactions must be reduced to a reasonable number.

The aggregation of data is inevitable if the theoretical model is to be made operational; general principles of aggregation are devised in order to solve this problem. The method of aggregation used will depend on the objectives of the study. Presently, there is no unified theory of aggregation. Several studies have been made in which an n by n matrix has been reduced to an m by m matrix. Predictions based on

the m by m matrix were then compared with the predictions based on the n by n matrix. These comparisons have lead to some general principles of aggregation. Although advanced mathematics has been used in formulating many of the principles, the basic criteria from which the principles are derived are frequently subjective.

When speaking of the limitations on the interindustry model, the major source of model error is usually attributed to the assumption of fixed activity coefficients. Unfortunately, this belief has limited the basic research done on the aggregation problem. It is the belief of the author that the aggregation problem at minimum, contributes a source of error comparable to the one introduced by the fixed activity coefficient assumption. Furthermore, the problem is of greater significance in the long-run.

In the short-run, even with technological change, the activity coefficients show signs of relative stability. Also, as the technical information on production functions of key industries becomes more abundant, the change in activity coefficients will be predicted with greater accuracy. Furthermore, statistical techniques may be used to estimate the direct inputs in order to calculate the activity coefficients for the major industries. Conditions such as these reduce the limitations on the model due to the fixed coefficient assumption.

The aggregation problem is of extreme importance in both the short- and the long-run and is one of the basic problems of interindustry model construction. The construction of a transactions matrix.

is always related to the problem of aggregation. An n by n transactions matrix represents the aggregation of the transactions of a countless number of industries. The current problem is to establish some general principles for aggregating the industries in the model.

Some General Principles of Aggregation

A survey of the literature reveals that the problem of aggregation is not only unsolved, but is receiving little attention in an attempt to obtain a solution. There are a few articles written on some highly specialized topics of aggregation theory. However, there are very few principles of aggregation theory. Several authors, Balderston,² Whitin, and Theil³, are working towards a general theory of aggregation. Unfortunately, the majority of their constraints are defined by equations and must be translated into a verbal statement in order that the practitioner of interindustry analysis may use their results. This section provides a summary of their work in the form of verbal statements. Incorporated in this discussion is a statement of several principles which are given in mathematical form in several recent articles on aggregation theory.

Homogeneous Output.-- The industries which are to be aggregated should have almost perfectly homogeneous output. Practically, it is nearly impossible to obtain a perfectly homogeneous output for each

²O. Morgenstern, Economic Activity Analysis (New York: John Wiley, 1954). p. 79.

³H. Theil, "Linear Aggregation in Input-Output Analysis," Econometrica, XXV (1957), 111-122.

industry within the model. There is a definitional problem in determining the exact meaning of homogeneous output. One of the criteria to be followed is that of substitutability. If the industry products are close substitutes, as viewed by the user, the outputs of each industry may be classified as homogeneous outputs. These definitional problems illustrate the subjective nature of the present aggregation theory.

Closely related to the idea of homogeneous outputs is the concept of the similarity of inputs. Industries may be aggregated with this criterion as the basic frame of reference. Unfortunately, there is often a conflict between the two criteria of homogeneous output and similarity of inputs. In the event of such a dilemma the final decision is subjective and depends on the objectives of the study. However, in the opinion of the author, it is generally preferable to aggregate industries using the criterion of homogeneous output rather than that of similarity of inputs.

Perfect Proportion in the Product Mix.-- Closely related to the concept of homogeneous output is the proportion of different products in the industry output. Since the aggregation procedure combines the output of several industries into one industry, the output is not perfectly homogeneous even though the products may be close substitutes for each other. However, this difficulty is minimized if the relative positions of each product remain constant in the aggregate figure or change by a fixed pattern.

For example, consider the chemical industry in a regional analysis. The aggregated industry output may include the output of synthetic fiber firms in addition to the output of basic chemicals. Therefore, any output predictions generated from the model will have to be broken down by some known scaling factor in order to determine the output of the synthetic fiber industry. Although the assumption of a fixed proportion of total output leads to the simplest form of empirical analysis, other projections may be made in order to determine the new proportions in the product mix.

Exclusive Use.-- If the output of one industry is used exclusively as input to another industry, the two industries can be combined without introducing any serious empirical errors in the model. This principle of exclusive use is a valuable tool in making decisions regarding the vertical or horizontal aggregation of industries. Horizontal aggregation involves the combination of industries whose products are in the same stage of manufacturing whereas vertical aggregation involves a combination of all the successive stages of the manufacturing process.

For example, gasoline and other petroleum derivatives are all obtained from the same input but represent outputs from different stages of the cracking process. Should each of the different outputs be combined by horizontal aggregation thereby creating many industries or should they all be combined with another industry in the complex? The principle of exclusive use states that vertical aggregation is

permitted if the major part of the petroleum output goes to another industry in the complex. Such occurrences are common in local regional studies.

Similarity of Production Functions.-- One of the disadvantages in the aggregation of interindustry data is that the activity coefficients, which are an integral part of the production function, may show a large change when compared with the coefficients based on the data before they are aggregated. One method available for the solution of this problem involves the aggregation of industries according to the similarity of their production functions.⁴

The acceptance or rejection of these criteria is mainly subjective due to the absence of detailed empirical information on the nature of production functions. However, there is one serious criticism to this method of aggregation. A fundamental omission is that a great many technological changes are not merely changes in the production function, but in the product. It could probably be demonstrated that most technological changes are accompanied by some sort of change in product. The aggregation of industries by the nature of their production coefficients is closely related to the concept of horizontal aggregation. Horizontal aggregation, which involves the aggregation of products at the same stage of the manufacturing process, is related to aggregation by the nature of the production functions since products at the same stage of manufacturing process are likely to have similar production functions.

⁴O. Morgenstern, Economic Activity Analysis (New York: John Wiley, 1954), p. 96.

Complementarity.-- Complementarity pertains to the demand for the final products of each industry to be aggregated. If the demand for the products of two or more industries increases or decreases together, then the products are said to be complementary. If this condition exists the industries may be aggregated. One of the empirical disadvantages of this approach is the necessity for obtaining enough data to determine whether or not the products are complementary.

Minimal Distance Criterion.-- Professor Walter D. Fisher of Kansas State College has utilized some of the basic concepts of the analysis of variance in order to derive a criterion for aggregating industries. Overly optimistic, he states that

small industries in input-output tables can be aggregated into large industries on the basis of similarity of coefficients, stability of the inverse, homogeneity of industries, similarity of production functions, and similarity of the cost structure. The minimum distance idea seems to encompass all of these suggestions.⁵

$$\text{Let } C = \sum_{i=1}^m \sum_{j=1}^n (\bar{b}_{IJ} - b_{IJ})^2 \bar{y}_j$$

$$\text{Where } \bar{b}_{IJ} \text{ is the weighted mean } \frac{\sum_{j \in J} y_i b_{IJ}}{\sum_{j \in J} \bar{y}_j}$$

b_{IJ} is the element of the aggregate Leontief inverse matrix

$$\left[I - (a_{IJ}) \right]^{-1} \text{ that corresponds in position with } a_{IJ},$$

$$b_{IJ} = \sum_{i \in I} b_{ij}$$

⁵W. D. Fisher, "Criteria for Aggregation in Input-Output Analysis," The Review of Economics and Statistics, XL (1950), 250-260.

where b_{ij} is the element of the Leontief inverse

$$\text{matrix } [I - (a_{ij})]^{-1}$$

corresponding to a_{ij} , \bar{Y}_j is the final demand for industry j in the base year, and n and m represent the number of industries before and after aggregation.

Physically, C can be regarded as the deviations around a line. Therefore minimizing C may be interpreted as minimizing a distance, hence leading to the name minimal distance criterion.

There is one serious disadvantage in this approach. The derivation of the minimal distance criterion assumed that $\text{cov}_{jk} = 0$ for all j and k . Such an assumption presents no mathematical difficulties; however, the economic interpretations of this assumption are quite restrictive. If the co-variance is zero, it is impossible to consider the possibility of complementarity between the final demands of each industry to be aggregated.

The Acceptability of Aggregation

There are many standards which can be used to determine if a given method of aggregation is acceptable. One of the most promising is the one suggested by Professor M. Hatanaka. He states, "One may consider that a consolidation is acceptable if the consolidated final demand corresponds to the consolidated net outputs for the final demand."⁶ This statement can be translated into a mathematical identity.

⁶M. Hatanaka, Testing the Workability of Input-Output Analysis, Economic Research Report (Princeton, N.J.: Princeton University, 1957), p. 301.

Let Y and X represent the final demand and the net outputs of the n sectors before aggregation with activity coefficients a_{ij} . Let R and S represent the final demand and the net outputs of m sectors after aggregation with activity coefficients b_{ij} .

Therefore

$$(I-A)X = Y$$

$$(I-B)S = R$$

Where

$$A = (a_{ij})$$

$$B = (b_{ij})$$

In the proposed method of aggregation let the first $t(1)$ sectors be aggregated into the 1st group, the second $t(2)$ sectors be aggregated into the 2nd group and the last $t(m)$ sectors be aggregated into the m th group.⁷ Therefore, this method of aggregation is defined by the following matrix:

$$T = \begin{bmatrix} e(1) & 0 & 0 & 0 \\ 0 & e(2) & 0 & 0 \\ . & . & . & . \\ 0 & 0 & 0 & e(m) \end{bmatrix}$$

where $e(i) = (1, 1, \dots, 1)$, $(i = 1, 2, \dots, m)$

Hence

$$R = TY$$

$$S = TX$$

⁷K. Ara, "The Aggregation Problem in Input-Output Analysis," Econometrica, XXVII (1959), 257-262.

Therefore

$$(I-B)S = (I-B)TX = R = TY = T(I-A)X$$

$$(I-B)TX = T(I-A)X$$

It follows that

$$BT = TA$$

Thus far the analysis is merely a rigorous statement of a criterion for the acceptability of aggregation. However, there are no operational constraints on the original matrix A . Ara proved⁸ that it is necessary and sufficient for the aggregation of sectors to be acceptable if the total sums of each columns elements in respective submatrices A_{ij} be equal, where A_{ij} is the submatrix of $t(i)$ rows and $t(j)$ columns in A .

7.3 Statistical Error

Statistical error, as was model error, can be traced back to the basic assumptions made in the formulation of the model. Statistical error usually results from inaccurate observations of economic phenomena. Frequently these inaccuracies are a result of approximating nonexistent data. Profesaor John M. Ryan of the University of North Carolina states that

the basic data on inputs and outputs by industries are frequently faulty and often nonexistent. As a result the productivity coefficients computed from these data are in reality only estimates of the actual coefficients. It is impossible to obtain with any accuracy the distribution of the estimates about the actual values. There

⁸Ibid., p 260.

is certainly no reason to assume that the distribution is normal. Without knowing the distributions it is impossible to assign bounds to errors arising from the use of these estimates. Thus, it is clear that using available data to estimate the productivity coefficients introduced a 'statistical error' of unknown magnitude.⁹

In this discussion, statistical error results from the following problems: operational definitions, measurement, and statistical imputation.

Operational Definitions

The greatest definitional problem in interindustry analysis involves the definition of an industry. In general, industries in the manufacturing sector of the model are defined as a group of semi-homogeneous establishments. However, this definition leads to several problems. Many establishments are actually corporations which are involved in many different manufacturing activities. If each of the different manufacturing divisions were under separate ownership, then each of these different divisions might be classified as an industry and not combined into one corporation.

An examination of the definition of the word industry for the non-manufacturing sectors of the model, such as the agriculture and service sectors, leads to additional definitional problems. The output of the agriculture sector is not collected on an industry basis but on a commodity basis. The output of the service industry is defined in terms of the distributive or service functions it performs. The definitional problems are significant because of the way they affect the measurement of gross output in each of the industries.

⁹J. M. Ryan, "The Leontief System," Southern Economic Journal (1953), p. 491.

The following example illustrates the problems arising from the definition of industry.¹⁰ Let there be two industries producing both a primary product and a secondary product. The primary products of industry 1 and industry 2 are X_1 and W_2 . The secondary products of each industry are W_1 and X_2 , respectively.

Therefore, if the outputs for the input-output table are calculated on an industry basis, the output of industries 1 and 2 would be given by $X_1 + W_1$ and $X_2 + W_2$, respectively.

If the product definition of industry were used, the output of industry 1 would be $X_1 + X_2$ and the output of industry 2 would be $W_1 + W_2$.

There are disadvantages in both of the above definitions. If the first definition were used, the output requirements generated by the model would apply only to the primary product with complete neglect of the secondary product. The difficulty encountered in using a product definition of an industry is that it removes all relationships between establishment identity and its corresponding products which are essential in mobilization analysis.

The definition of industry used for the 1947 table was a compromise of the industry and product definitions. The output of industry 1 is defined as $X_1 + W_1 + X_2$; the output of industry 2 is defined as $X_2 + W_2 + W_1$. Consider the output of industry 1, $X_1 + W_1 + X_2$. Since the outputs are associative, it may be written as $X_1 + X_2 + W_1$. Therefore, the output of industry 1 is equal to the sum of its primary output (X_1),

¹⁰National Bureau of Economic Research, Input-Output Analysis: An Appraisal, Conference on Research and Wealth (Princeton, N.J.: Princeton University Press, 1955) p. 236.

the secondary output of industry 2 (X_2), and the secondary output of industry 1 (W_1). Similarly, the output of industry 2 is the sum of its primary output (W_2), the secondary output of industry 1 (W_1), and the secondary output of industry 2 (X_2).

Measurement

Measurement or the observation of economic data is another source of statistical error. The question to be answered is how accurate are the entries in the interindustry table. If the agricultural industry sells X dollars of its output to the chemical industry, how accurate is this figure? The accuracy of these interindustry flows is of extreme importance for several reasons.

First, the transactions table is the most basic element in the interindustry model. From these flows, the activity coefficients are calculated which then form the basis for the calculation of the interdependence coefficients. Once the interdependence coefficients are known, the model can be used for economic forecasting.

Second, the transactions table is the basis of structural analyses which involve a study of the properties of the economic model without any reference to its use in making predictions. The transactions table, as such, contains no hypotheses concerning the activities in any complex; it is merely an accounting device used for the classification of interindustry data. Each activity coefficient which is obtained from the transactions table represents a structural parameter which describes the nature of the interindustry transactions in an economy.

The above discussion emphasizes the advantages in obtaining accurate interindustry data even though the model may not be used for predictive purposes. The remainder of this section contains a discussion of the major sources of measurement error. The topics discussed are those sources of measurement error which are found most frequently in interindustry analysis. For a more general analysis consult Morgenstern's, On the Accuracy of Economic Observations.¹¹

Lack of Experimental Design.-- The data used in many economic studies are not obtained from a planned experiment. Usually the data are collected by some agency which has no specific experiment or objective to be accomplished. At a later date these statistics may form the bases for an economic study. However, since they are accumulated, sorted, and classified without any particular purpose in order to be applicable to many economic studies, they may not be exactly suited for many specific studies. Unfortunately, the social scientist is not in the enviable position of his colleague, the physical scientist, since the social scientist cannot obtain socio-economic data under the controlled conditions of a laboratory. Therefore, the social scientist cannot always design his experiment in order to obtain the most relevant information for his study. He is forced to use existing data which may not be exactly suited to his needs. The use of such data introduces statistical error which may be attributed to the lack of designed experiments.

¹¹O. Morgenstern, On the Accuracy of Economic Observations (Princeton, N. J.: Princeton University Press, 1950).

Another problem which is related to the lack of experimental design is the different educational backgrounds of the people engaged in the compilation of the statistics. This is of particular interest in the use of questionnaires. Many observers are chosen on an ad hoc basis and given only a limited amount of training. Each person will have different interpretations of many of the terms in the questionnaire. The end result will be an inconsistency in the data.

Faulty Design of Questionnaires.-- The questionnaire is one of the primary tools used in the acquiring of interindustry data. In particular, it is used to a very large extent in regional studies. The statistical theory underlying the design of questionnaires is meager. However, there are several obvious characteristics which should be common to all questionnaires.

First, each term in the questionnaire should have a precise meaning in order to lead to unique, unambiguous answers.

Second, each respondent should be informed of the purpose of the study.

Third, each questionnaire should include a complete list of instructions.

In many interindustry studies, it is impossible to contact all the sources of relevant data; therefore, a sample is drawn. Sampling theory is developed to a higher degree than is the present theory regarding the design of questionnaires. However, even though the statistician has provided the social scientist with some basic rules for the

design of sampling plans, the social scientist has not used them to their fullest extent. The importance of an accurate sampling plan can not be over-emphasized in interindustry analysis. An important problem in interindustry studies is the estimation of the gross output for each sector in the model. The most accurate and best designed questionnaire gives rise to erroneous gross output figures (statistical error) if the sampling plan is inaccurate.

The Non-Reproducibility of Economic Data.-- Once observed, economic data are unique since they cannot be reproduced. This non-reproducibility of economic data would not contribute to any known source of statistical error in interindustry analysis if each observer arrives at the same value for every economic observation. Unfortunately, many observers obtain different values for the same economic observation. On occasion, the differences can be reconciled depending on the time interval between the initial observation and its publication. However, the reconciliation will not necessarily be correct since it is impossible to duplicate all the conditions which were in existence at the time of the original observation. These reconciliations usually result in a compromise between the various observations; such reconciliations are another source of statistical error in the collection of interindustry data.

These differences in the observation of economic data and their corresponding reconciliations are inevitable; however, econometrics is in need of more refined methods for the acceptance of the reconciliations. Morgenstern states that

It would be excellent if economists, at an early time in their training and careers, would become acquainted with work showing them the extreme trouble to which physicists must go in order to be able to accept data. Similar quantitative work ought, of course, be done in economics. When comparable standards have become the rule in economic research, economics will be placed on surer ground, while at the same time its extraordinary difficulties will be better appreciated.¹²

Another factor contributing to the non-reproducibility of economic data is the freedom of private enterprise not to offer full disclosure of the relevant data requested in a questionnaire. Some firms may fail to give accurate responses for a variety of reasons some of which are: tax evasion, distrust of the agency requesting the information, and failure to recognize the need for the study thereby furnishing inaccurate data.

Statistical Imputation

Statistical imputation or the estimation of non-accessible interindustry data is another source of statistical error in interindustry models. The need for statistical imputation arises from the inadequacy of the data describing interindustry transactions. However, even with significant advances in the data collection procedures, the problem will always exist if a high degree of statistical refinement is required in the model.

Traditionally, statistical imputation has been accomplished by historical relationships and secondary data sources. Historical relationships, which often become rules of thumb, lead to error in the data since they are not usually constant through time. However, these

¹²Ibid., p. 20

historical relationships may lead to accurate estimates in a regional analysis if those persons using them are very familiar with the given complex.

Secondary data sources, similar to those used in Chapter 3, provide another method of statistical imputation and lead to another source of statistical error. Secondary data sources involve the use of supplementary relationships, i.e., economic relationships which are not contained in the model. Chapter 3 contains an analysis of the way in which secondary data sources lead to statistical error.

There is one special aspect of statistical imputation which deserves particular attention, i.e., the possible error in the inverse resulting from zeros in the off-diagonal entries. In this case, the interindustry flows between some sectors may be so small that they are neglected, i.e., set equal to zero and not subjected to the techniques of statistical imputation. This operation has a significant effect on the noise in the inverse, i.e., the rounding errors which are involved in large scale computations. As the number of zeros in the off-diagonal elements of the matrix increase, the noise is reduced and hence the matrix will show a higher degree of stability. However, if these zeros are supposed to be small numbers, the noise of the inversion will increase and its stability will decrease. An improvement in the methods of statistical computation will provide a partial solution of this problem.

7.4 Computational Error

In interindustry analysis, computational error results from the numerical solution of large equation systems. It is practically impossible to eliminate computational error although it is possible to reduce it by further mathematical investigations into the nature and characteristics of the Leontief matrix. Even if computational error is reduced, there should be quantitative estimates of these errors. The estimates of these computational errors are of extreme importance since they affect the predicted levels of output for any given final demand.

Walras and Pareto were not concerned with computational errors. They merely counted the number of equations in a model and compared this number with the number of variables; if both quantities were equal, they assumed that a numerical solution was possible. As shown in the previous chapters, an identity in the number of unknowns and equations in a model is neither a necessary nor sufficient condition for its solvability. Therefore, present day economists must not only formulate their model in terms of equations, but in order for their system to be economically meaningful, they must show that a solution exists.

The number of two-factor multiplications involved in obtaining the inverse of a matrix with n rows and columns is approximately n^3 , thus for a 38 by 38 inverse about 50,000 multiplications are required; for a 190 by 190, about 7,000,000 and for a 450 by 450 about 90,000,000.¹³ With such a large number of calculations, small errors, such as rounding, may accumulate until the error in the final result is very large.

¹³National Bureau of Economic Research, Input-Output Analysis: An Appraisal, Conference on Research and Wealth (Princeton, N. J.: Princeton University Press, 1955), p. 145.

Very small numerical errors in the data often result in large errors even in the simple case of two simultaneous equations. Morgenstern offers the following example taken from Milne:¹⁴

The equations

$$x - y = 1$$

$$x - 1.00001y = 0$$

have the solution

$$x = 100,001 \quad y = 100,000$$

while the equations

$$x - y = 1$$

$$x - .99999y = 0 \quad \text{have the solution}$$

$$x = -99,999 \quad y = -100,000$$

The coefficients in the two sets of equations differ by at most two units in the fifth decimal place, yet the solutions differ by 200000.

The previous examples illustrate the importance of computational error. If econometric models are to be made operational, the errors due to computation must be estimated. No longer can be economist merely formulate a model and state that its solution has great economic significance unless he states the conditions for its solvability. Pareto stated that if his theory were applied to 100 persons exchanging 700 commodities among each other, one would obtain 70,699 equations.¹⁵ Models of this type are of interest from a methodological

¹⁴O. Morgenstern, Economic Activity Analysis (New York: John Wiley, 1954), p. 490.

¹⁵Ibid., p. 490.

standpoint but if the discipline (marketing) is to make any predictions it must resort to empirical analysis. At the time of its conception, Pareto's model could not be solved due to its high order; today, even with the use of electronic computers, the model still cannot be solved.

The remainder of this section contains a description of the different types of computational error introduced into an interindustry analysis due to the solving of high order equation systems. The discussion is not entirely exhaustive in that it does not contain a description of all the types of computational error; however, it does consider the major components of computational error.

Noise

Noise is the operational interference introduced into interindustry calculations by the presence of an enormous number of operations. One of the greatest contributions to noise results from the rounding process. Many electronic computers can carry only eight significant digits; therefore all others are dropped and the eighth digit is changed accordingly. If the inverse of a 38 by 38 matrix requires 50,000 multiplications, considerable error may be introduced due to the accumulation of these rounding errors.

If these rounding errors become critical, the model will generate nonsensical results even though all the input data are quite accurate and the model is represented by a solvable system of equations. Unfortunately, those engaged in the formulation of abstract economic models feel that problems of this nature are of minor importance. The

relationship between noise, solvability, and accurate input data, is a major problem confronting interindustry theorists.

Rounding errors are introduced by subtracting two numbers which have a large number of significant figures. The result may be a number with only a few significant digits as compared to the two original numbers. In these cases, rounding has a large effect on the final number. One possible solution to this problem is to use the Gauss-Seidel iteration for solving for the outputs of each industry.¹⁶ The iterative process is as follows: Let $X_i^{(m)}$ denote the m th estimate of the value of X for the i th sector. Therefore

$$X_i^{(m)} = Y_i + a_{i1}X_1^{(m-1)} + a_{i2}X_2^{(m-1)} + \dots + a_{in}X_n^{(m-1)}$$

Obviously the use of such iterative procedures restricts the magnitude of the rounding errors since there are no subtractions involved.

Noise is not limited to rounding errors. It is true that rounding errors are the greatest contributors to noise in digital computer calculations; however, it is also present in the use of analogue computers. Analogue computers attempt to simulate a mathematical constraint by the use of some corresponding physical analogy. Since there is always an imperfection introduced by the embodiment of some mathematical principle into a physical situation, analogue computers generate noise.

¹⁶T. Barna (ed.), The Structural Interdependence of the Economy, Proceedings of an International Conference on Input-Output Analysis. (New York: John Wiley, 1954), p. 74.

Truncation Error

Truncation error is a result of terminating infinite processes in a finite number of steps. For example,

$$(I-A)^{-1} = I + A^1 + A^2 + \dots + A^n$$

The above is an infinite series. Therefore it would take infinite time to make all the calculations so designated by the series. Since the calculation must be completed in a finite time, the series must be stopped or truncated after a finite number of terms are calculated. Since the series is not carried out to the last term, the truncation process leads to another source of statistical error.

Usually, truncation of the Leontief inverse does not lead to enormous computational errors. Since the matrix has certain mathematical properties, such as $\sum_i a_{ij} \leq 1$, $a_{ij} \geq 0$, it is well behaved. Therefore, the first few terms of the progression may be used as an approximation of $(I-A)^{-1}$. Unlike the error due to noise, the truncation error can be estimated since $(I-A)(I-A)^{-1} = I$. This error is of no significance in either the 38 by 38, 1939 matrix nor the 44 by 44, 1947 matrix.

Truncation error is not limited particularly to the Leontief inverse. As the model becomes more refined, transcendental functions, e.g., e^x , are used for the solution of the dynamic case. In order to evaluate these functions, approximations are also made by truncating an infinite series.

Specious Accuracy

Specious accuracy results from providing numerical answers with too many decimal points which the input data do not warrant. An example of specious accuracy is given by the following example. The ratio of two numbers a and b , each of which has two significant digits, might be written to six point decimal accuracy. In such a case the result is very misleading for several reasons.

First, the basic input data in the model may not be known to such a high degree of accuracy. Therefore, from a statistical standpoint the additional decimal places in any given number are not only misleading but have no statistical foundation from the standpoint of numerical analysis.

Second, even if the input data justify such a high degree of numerical accuracy, the results may still be misleading since they imply that the model can generate solution sets with a high degree of accuracy. This is a completely erroneous assumption in the case of input-output models. A model which is formulated upon the assumptions of linear homogeneous production functions and homogeneous outputs, not to mention the problems of aggregation and computational error, is not likely to predict output levels to six decimal places. However, specious accuracy should not be confused with carrying a large number of decimal places in the solution of simultaneous equation systems. As previously discussed, there is a great advantage in carrying these intermediate calculations to a high degree of decimal accuracy since they limit the magnitude of the rounding error.

Evans has made an exhaustive study of the effects of structural errors on inter-industry projections.¹⁷ His conclusions are as follows:

1. With reasonable care in the construction of base tables, especially with respect to the autonomous and total activity vectors, interindustry estimates can be made with confidence that errors in structural matrices are not only noncumulative but rather compensating in effect.
2. Many of the error estimating properties of the approach are enhanced if the autonomous demands, factor payments, and activity totals in a base table are well established; hence special efforts to insure the accuracy of these controls are well worthwhile.
3. Both mathematical considerations and experience show that rounding errors in computations can be written off as a problem in this field.
4. Proportionality assumptions, in place of somewhat more realistic nonhomogeneous linear interrelationship functions may yield quite adequate results over a fairly wide range of problems if used with reason and care. The extensive work required to quantify nonproportional forms may properly be limited to cases where there is reason to suspect a significant departure from the simple ratio form and where the sector involved is directly important to a problem.
5. The many ways in which errors in quantification or formulation compensate should not lead to any inference that careful and detailed statistical work in establishing a structural matrix is not justified; on the contrary, confidence in the validity and usefulness of interindustry results must rest firmly on such a base. The data problem is the outstanding difficulty in this field today.
6. An important characteristic of the interindustry approach and a major contribution by it to economic research is the method's inherent ability to minimize undesirable effects from the errors which, because of imperfections throughout the body of available data, are sure to infiltrate the most careful economic analysis.

¹⁷W. D. Evans, "The Effect of Structural Matrix Errors on Interindustry Relations Matrix," Econometrica, XXII (1954) p. 479.

7.5 The Effect of Model, Statistical, and Computational Errors on Interindustry Projections

Previous discussions contain an analysis of the theory, use, and limitations of the interindustry approach. The object of this section is to examine the effects of the limitations of the model in its empirical use. A real test for any econometric model is to determine the validity of its results by comparing them with observed economic phenomena. This section describes the results of several of the empirical tests of the interindustry model. The majority of the tests involve a comparison of the output projections generated by the model with the actual outputs for a given period.

The validity of the model might be determined by comparing its predicted outputs with actual observed outputs for a given year. However, such a procedure does not give any insight into the usefulness of the model. In order to determine the usefulness of the model, the results generated by the model must be compared with alternative methods of predicting the levels of output. If interindustry analysis is to be a useful tool of empirical economic analysis it should show considerable improvement over the more elementary methods of estimating industrial output such as simple regression and a moderate improvement over the more advanced methods such as the multiple-correlation of time series.

Chenery¹⁸ offers the following restraints on the above procedures:

1. In making an input-output projection two methodologically different steps are involved, estimating the final demands on all industries and estimating the total production required in all industries to meet both these final demands and the intermediate demands from other industries. Only the latter step is distinctive of input-output analysis and therefore the overall tests should if possible be artificial projections based on actual final demands and actual total outputs in a past year.
2. The input-output technique ought to be most useful in analyzing the effects of radical changes in the composition of final demands. In the extreme case in which all final demands changed proportionately, it would project proportional changes in all outputs and the technique is hardly necessary for such a simple projection. In cases of radical change in final demands, on the other hand, as in war mobilization or in forced-draft economic development, alternative techniques based on continuation of the past run into difficulty, and the input-output technique ought to come into its own. Unfortunately, the overall tests which have been made all refer to peacetime years which do not have such radical changes in the composition of final demands.
3. The provision of reliable data in actual final demands and actual total outputs, consistent in form with input-output concepts, is a difficult task in its own right. Therefore, it is possible for the output projections to show a different degree of accuracy than is shown by comparing output projection with actual observed levels.

The remainder of this section contains a description of the major empirical investigations regarding the accuracy of Leontief's model. The implications of these tests are discussed in the next chapter.

¹⁸H. B. Chenery and P. G. Clark, Interindustry Economics (New York: John Wiley, 1959), p. 165.

The majority of the early testing of the model was performed by Leontief. His empirical tests fall into three distinct categories which are described in the following paragraphs.

1929 Backcast

In this experiment, Leontief used the data from the 1939 Census of Manufacturers to derive a transactions table which in turn lead to the computation of a table of activity coefficients for 1939. Next he determined the actual bill of goods for 1929 from pertinent data sources. Given the activity coefficients and the bill of goods he predicted the level of outputs corresponding to the 1929 bill of goods by the use of the interindustry model. The results of his calculations are given in Table 7.5-1.

The accuracy of the predictions is surprising for several reasons. First, the activity coefficients were applied to data which were ten years old. Although a basic assumption of the model is that the coefficients are invariant, they are seldom used for predictions of such a long time interval without some minor modification.

Second, the entire economy was aggregated into 10 industries. The previous discussion concerning the problem of aggregation emphasizes the discrepancies introduced by such a large degree of aggregation. In fact the aggregation used in this analysis is 50 times as great as that used in some of the later verifications of the model.

TABLE 7.5-1--A COMPARISON OF THE ACTUAL AND INDIRECT DEMANDS FOR 1929 AND OF ESTIMATES BASED ON THE
STRUCTURE OF 1939
(Millions of Dollars; 1939 Prices)

Industry	(1) Bill of goods	(2) 1929 Output	(3) 1929 Output Estimate	(4) Actual Direct Demand	(5) Estimated Indirect Demand	(6) Errors in Estimating Indirect Demand	(7) Error as Percent of actual
				(2-1)	(3-1)	(5-4)	(6/4)
Agriculture	9227	11496	11512	2269	2285	16	+0.7
Minerals	143	3711	3647	3568	3504	-64	-1.8
Metal Fabricating	5029	15909	13964	10880	8935	-1945	-17.9
Fuel and Power	3998	8822	8992	4824	4994	170	+3.5
Textiles, leather and rubber	6009	7677	7465	1668	1456	-212	-12.7
Railroad Transport	812	5669	4081	4887	3269	-1618	-33.1
Foreign Trade	619	3673	3115	3054	2496	-558	-18.3
Industries, n.e.c.	9555	19003	20972	9448	11417	1969	+20.8
Government and other industries	23346	48836	52563	25490	29217	3727	+14.6

Source: O. Morgenstern, Economic Activity Analysis. (New York: John Wiley, 1954), p. 60

Standard Errors of Prediction for Various
Methods of Output Projection

Leontief states that

this test has been designed to compare the magnitude of error resulting from a purely mechanical application of the input-output technique, with assumption of constant input coefficients, to prediction of unknown total outputs of individual industries from a known bill of final demand with the error resulting from a similarly mechanical application to the same data of the more conventional methods of prediction currently in use. The comparison of the discrepancy between the actual 1919 and 1929 outputs and the outputs predicted for these years is summarized in the following table. It seems to point toward unmistakable superiority of the input-output technique at least in this particular case.¹⁹

TABLE 7.5-2--STANDARD ERRORS OF PREDICTION OF THIRTEEN INDUSTRY OUTPUTS
IN 1919 AND 1929 FROM 1939 DATA
(Millions of Dollars)

	1919	1929
METHOD I ^a	380	237
METHOD II ^b	1363	1744
METHOD III ^c	2021	1539

Source: W. Leontief, The Structure of the American Economy 1919-1929 (Cambridge, Mass.: Harvard University Press, 1951), p. 216.

^aInput-output.

^b $\frac{\text{Output of Industry}}{\text{Gross National Product}} = \text{Constant}$

^c $\frac{\text{Output of Industry}}{\text{Final Demand of Industry}} = \text{Constant}$

¹⁹W. Leontief, The Structure of the American Economy, 1919-1929 (Cambridge, Mass.: Harvard University Press, 1951), p. 216.

The Variation of Activity Coefficients Between 1919 and 1939

In this study Leontief calculated the activity coefficients for 1919, 1929, and 1939. He then made an exhaustive statistical analysis of the nature and significance of the change in these activity coefficients. The results of this study are contained in Studies and are not summarized in this thesis.²⁰ However, the following table indicates some of the major results of his analysis.

TABLE 7.5-3--STATISTICAL CHARACTERISTICS OF CHANGES IN ACTIVITY COEFFICIENTS

	1919-1929 Distribution	1929-1939 Distribution
Mean		
All Coefficients	-.14	-.14
All Non-labor Coefficients	.60	.30
Labor Coefficients Only	-.36	-.16
Standard Deviation of All Coefficients	.35	.35

Source: W. Leontief, Studies in the Structure of the American Economy. (New York: Oxford University Press, 1953), p. 28.

The BLS Test, 1929 to 1937

Hoffenberg did one of the most extensive overall tests of the Leontief model. Given the 1939 38 by 38 matrix and the final demands for each industry in 1929, 1931, 1933, 1935, and 1937, he calculated

²⁰W. Leontief, Studies in the Structure of the American Economy. (New York: Oxford University Press, 1953), p. 17.

the corresponding output for each year. He then compared the output projections of the model with the final demand and Gross National Product projections. The techniques of these alternate types of projections are given in Table 7.5-4.

TABLE 7.5-4--AVERAGE ERRORS OF ALTERNATIVE PROJECTIONS 1929-1937
(Billions of Dollars)

	1929	1931	1933	1935	1937	1929-1937 Average
Percentage Errors						
Input-output	18	17	13	7	5	12
Final Demand	19	24	14	8	5	24
GNP	23	26	35	10	10	21
Algebraic Averages						
Input-Output	7	11	6	0	-4	5
Final Demand	14	20	5	1	-4	7
GNP	1	20	24	11	-8	8

Source: National Bureau of Economic Research. Input-Output Analysis: An Appraisal. Conference on Research and Wealth (Princeton, N. J.: Princeton University Press 1955), p. 163; H. B. Chenery and P. G. Clark, Interindustry Economics (New York: John Wiley, 1959), p. 167.

Barnett's Test 1950 Full Employment Projection

The previous tests involved backward predictions of output, i.e., they started with a known final demand and calculated the

corresponding output levels. In 1947, Barnett used the model for one of its actual purposes, i.e., forecasting future outputs needed for full employment. The primary difference between this test and the previous tests is that in this test the final demand had to be predetermined before the output levels could be forecast.

Barnett also increased the scope of the analysis. The alternative tests of Leontief and Hoffenberg are statistically naive since they merely applied constants to output data. In addition to using these elementary projection methods, Barnett used a fourth method in which he used linear multiple regression analysis based on the data for the years 1922 to 1941, and 1946. Time and GNP were the independent variables. His analysis was based on two alternative hypothetical models. In the first, the Consumption Model, full employment was attained solely by expanded consumption and in the second, the Investment Model, full employment was obtained solely by expanded investment. The results of Barnett's test are given in the following table:

TABLE 7.5-5--AVERAGE ERRORS OF ALTERNATIVE PROJECTIONS, 1950
(Billions of Dollars)

	Index Points		Absolute Value	
	Consumption Model	Investment Model	Consumption Model	Investment Model
Input-Output	32	35	877	926
Multiple Regression	42	30	662	424
Final Demand	34	39	975	1030
GNP	50	50	1115	1049

Source: H. B. Chenery and P. G. Clark, Interindustry Economics (New York: John Wiley, 1959), p. 170.

Ghosh's Use of Nonhomogeneous Production Functions

The theoretical advantages in using nonhomogeneous production functions or two-point estimates of activity coefficients are discussed earlier in this thesis. At that time, no empirical investigations were given to support the argument. Ghosh²¹ has made one of the first empirical extensions of the model.

The previous tests of Leontief, Hoffenberg, and Barnett, stayed within the confines of the basic model in that their analysis contained no deviation from the basic assumptions. Ghosh has modified the model by the use of two-point activity estimates. In general his results show that two-point estimates offer better output projections than those of the conventional Leontief type.

TABLE 7.5-6--OUTPUT ESTIMATES FOR 1939 and 1947
(Millions of Dollars, 1929)

	1939 Estimates			1947 Estimates		
	Leontief Matrix $y = bx$ 1929	Present Matrix $y = a+bx$ 1919-29	Actual 1939	Leontief Matrix $y = bx$ 1929	Present Matrix $y = a+bx$ 1929-39	Actual 1947
Agriculture	24380	23846	25131	30827	27209	29187
Mining	4333	4516	3920	7990	7964	7150
Industry	65932	69806	69806	120769	131192	150391
Fuel	10554	10885	10308	14365	14231	8402
Transport	7371	6537	5053	13968	10618	7987

Source: A. Ghosh, "A Note on the Leontief Model with Non-Homogeneous Production Functions," Metroeconomica, XII (1960), 19.

²¹A. Ghosh, "A Note on the Leontief Model with Non-Homogeneous Production Functions," Metroeconomica, XII (1960), 14-21.

CHAPTER 8

CONCLUSION: THE USE OF INTERINDUSTRY MODELS AS A STATEMENT OF FUNCTIONALISM

The use of interindustry models as a statement of functionalism in marketing is a particular answer to the more general problem of the applicability of linear economic models to marketing. The previous chapters contain an analysis of the theory underlying the construction of marketing models in addition to the use and limitations of one particular linear economic model, the interindustry model for the solution of marketing problems. However, none of these technical limitations of the interindustry model were related to their effects on the development of those marketing models which emphasize the functionalism approach in market analysis.

Functionalism represents a combination and an extension of the three traditional approaches to marketing, the commodity, institutional, and functional approaches. The relation of functionalism to interindustry analysis is discussed in the introduction and is not repeated in this chapter. The objective of this chapter is to show that economic models, such as the interindustry model, must undergo considerable modification before they can solve the practical problems which confront the market analyst.

8.1 Limitations on Functionalism Resulting From the Use of Interindustry Models

Non-Holistic Approach

The interindustry model is primarily concerned with economic phenomena. However, in order to aid in the solution of marketing problems, additional variables, other than those related to economic phenomena, should be considered. Therefore, in addition to the equations describing economic phenomena, other equations must be added to describe the results obtained from the other fields in the social sciences.

Social psychology, sociology, anthropology, and psychology are beginning to use quantitative techniques.¹ The results of these studies may lead to the development of non-economic behavioral equations which increase the realism of the model.

Constancy of Interregional Distribution Systems

If the interindustry model is to be applied to regional marketing analysis, each economic region must be treated as a separate economic sector. Not only are the technological relationships between industries assumed to be invariant but the model also assumes the distribution of sales between industries in a given region to be constant. In a dynamic market, such an assumption represents a serious restriction on marketing analysis.

The Absence of Non-Economic Behavioral Equations

In the opinion of the author, the greatest limitation of

¹J.C. Charlesworth, Mathematics and the Social Sciences (Philadelphia: The American Academy of Political and Social Sciences, 1963).

interindustry analysis as a basis for the construction of an operational statement of functionalism is the absence of non-economic behavioral equations. As previously discussed, behavioral equations describe the actions of the basic elements in any system. In an economic system these basic elements are households and business firms. A variety of equations can be written to describe the actions of these basic elements.

The interindustry model is nearly void of economic behavioral equations and completely void of non-economic constraints. Given a bill of goods and the activity coefficients for each industry the interindustry model predicts output levels. Given the same information, traditional economic analysis would not lead to a prediction of output levels unless some economic behavioral principle such as the maximization of profits by the individual firms were given. Therefore, the interindustry model appears to eliminate the problem of having adequate knowledge concerning the behavioral patterns of firms and households. However, the elimination of these economic and particularly the non-economic behavioral principles seriously restricts the use of interindustry models as an axiomatic statement of functionalism.

In interindustry models, the economic ends desired by an individual are taken as given data and no attempt is made to explain them. For example, "Psychology, physiology, cultural history, and many other disciplines may make it their business to investigate why men like to drink alcohol; in economics what is alone of importance is that a demand for alcoholic beverages exists in a definite volume and strength."²

²L. Von Mises, Epistemological Problems of Economics (Princeton, N. J.: D. Van Nostrand, 1960).

However, these underlying non-economic behavioral principles which give rise to the demand for an economic good are essential components of an axiomatic model of functionalism. For this reason, existing economic models, without extensive modification, will not provide an operational model of functionalism.

Concluding Statement

"The progress of economics has been and is so painfully slow because the concepts, the analytical tools, and the investigative tools employed by economists have been and are basically incompatible with the subject matter that economists study."³ To a large degree this criticism applies to marketing. However, the functionalism approach is a step towards matching the tools of market analysis with the current subject matter. Since functionalism is a new analytical tool in marketing, it is now in the conceptual stage and is not empirically operational. The following suggestions are offered so that the conceptual framework of functionalism can be used as an empirical tool of market analysis:

Modification of Economic Models.-- It is the opinion of the author that a quantitative model of functionalism should be constructed by modifying the more successful economic models. In particular, the interindustry model appears to be the most favorable economic model which could be modified in order to represent a model of functionalism.

³S.Schoeffler, The Failures of Economics: A Diagnostic Study (Cambridge, Mass.: Harvard University Press, 1955).

Plurality of models.- A variety of models should be used to describe different segments of the marketing process.

The Use of Behavioral Equations.- Behavioral equations which describe the actions of individuals and firms in the market should be incorporated into the model. These equations should be derived from a deductive-inductive examination of marketing phenomena.

The Construction of Isomorphic Models.-- The models should be structurally similar to the market phenomena they are to represent. The models must show a one-to-one correspondence between their components and the concepts which they are to represent. They must also contain a symbolic statement of the existing functional relationships between pertinent marketing variables.

Interdisciplinary Contributions.-- In addition to modifying existing economic models, the market analyst should also use the contributions of the other social sciences which are related to certain phases of the marketing process. The introduction of quantitative analysis is not limited to marketing and economics; many of the other social sciences such as sociology, psychology, political science, and anthropology also use quantitative models. The introduction of these models would be comparable with the plurality approach to marketing model construction.

For example, "sociology may describe the method in which institutions change; psychology would describe the modes of reaction one

might expect and how fast these change. The physical sciences and engineering would tell what the production functions are and what possibilities of change exist; and the geologist would contribute information on the existence and location of natural resources."⁴

The Limited Use of Natural Science Models.-- Generally the use of the models employed in the natural sciences will offer at best a very minor contribution to the development of marketing models. This criticism follows from the fact that the majority of the models in the natural sciences are mechanical models, i.e., for a given set of environmental conditions, the behavior of any element in the model is uniquely determined.

For example, in chemistry, a moles of reactant A and b moles of reactant B may be united so as to yield c moles of reactant C at a given temperature and pressure. However, a consumer with a given income and, for example, constant social and political conditions, may distribute his expenditures in an entirely different manner under the same environmental conditions. Therefore, the mechanistic models of the natural sciences have severe limitations when applied to marketing problems.

⁴Ibid., p. 85.

APPENDIX

A.1 Some Salient Characteristics of Input-Output Models

Chapter 2 discusses the construction of the basic model. In order that this model may be applied to a variety of problems, a critical examination of the properties of the model must be made. In the last decade, economists, mathematicians, and statisticians have done extensive research in this area; unfortunately, their results are published in many different books and journals, the majority of which use different notations.

A complete summary of these previous works is almost impossible for one writer to complete. This section contains those theorems and theoretical extensions of the model which appear to be most important for additional extensions of the model similar to the numerical extensions in this thesis. Therefore, the importance of the material included in this chapter represents a value judgment on the part of the author and should not be interpreted as the general opinion of all the practitioners of interindustry economics.

In obtaining the information for this section, two problems were encountered; first, the traditional problem of different notation; and second, the statement of the theorems was often quite general and not directly applicable to interindustry analysis. Therefore, the author took the liberty of changing the notation and on occasion the statement of the theorems in order to yield a consistent set of

properties. These changes may limit the scope of the original work; therefore, the reader should consult the original work if the present analysis appears to be incomplete. The proof of a theorem is included only if it provides additional explanatory information mandatory for a correct interpretation of the theorem.

Several parts of this section involve an explanation of the basic concepts of linear models and cannot be attributed to any specific writer. The major part of the discussion in this chapter involves a synthesis of existing theorems in order to form a basis for the examination of the more recent problems of input-output analysis.

Positive Equilibrium in the Leontief System

The object of this section is to examine the conditions under which the system is in a state of positive equilibrium. If the model is in a state of positive equilibrium, the output of all industries is greater than or equal to zero, which means that all the elements of $(I-a)^{-1}$ are non-negative. The subject of positive equilibrium is of extreme importance in the static case. A negative output for an industry, given a positive bill of goods, implies that the industry cannot keep itself in operation; however, if its output is positive, i.e., if it is in a state of positive equilibrium, then the industry has the ability to produce goods for final consumption in addition to keeping itself going.

The question of positive equilibrium centers around the properties of the inverse $(I-a)^{-1}$ which in turn describes the possibility of

obtaining solutions to the equations in the model. The equation system which describes the interindustry model has n equations in n unknowns. Usually such a system has a unique solution, but this is not always true. Furthermore, even if the system has a unique solution, the set must be non-negative (greater than or equal to zero) to have a meaningful economic interpretation.

In the preceding section the equations are assumed to be independent and consistent; this guarantees a unique solution to the set. This argument is now explained in detail. Another way of saying that n equations in n unknowns have a unique solution is to state that the

$$\text{rank } (\underline{c}) = n.$$

The rank of \underline{c} is defined as the number of linearly independent columns of \underline{c} . The set of vectors c_1, c_2, \dots, c_n are said to be linearly dependent if for some set of scalars k_i , not all of which are zero, there exists

$$\sum_{i=1}^n k_i c_i = 0.$$

If the vectors are not linearly dependent they are linearly independent. The square matrix \underline{c} is nonsingular if and only if its columns are linearly independent. Therefore, from the previous assumption, $|\underline{c}| \neq 0$. Only nonsingular matrices have inverses; therefore the square matrix \underline{c} has a unique inverse if it exists. The inverse exists since the equations are also assumed to be consistent. This analysis explains the reasons for assuming the equations to be independent and consistent but it does not describe under what conditions the assumption is correct.

The primary objective of this section is to determine the feasibility of producing a positive bill of goods, i.e., to determine the requirements for positive equilibrium which imply that no industry produces a negative output. Several writers have solved the problem, given certain restraints on a_{ij} . The outcome of the Metzler-Hawkins-Simons theorems is that for a non-negative matrix \underline{a} , $(I-\underline{a})^{-1}$ will have no negative elements if and only if all the characteristic roots of \underline{a} are less than one in absolute value.¹

The Metzler-Hawkins-Simons condition also implies that all of the principal minors of $(I-\underline{a})$ are positive. The naturally ordered principal minors of $(I-\underline{a})$ are indicated by the dashed lines:

$$\begin{array}{ccccc} \boxed{1-a_{11}} & -a_{12} & -a_{13} & \cdot & -a_{1n} \\ -a_{21} & \boxed{1-a_{22}} & -a_{23} & \cdot & -a_{2n} \\ -a_{31} & -a_{32} & \boxed{1-a_{33}} & \cdot & -a_{3n} \\ -a_{41} & -a_{42} & -a_{43} & \cdot & -a_{4n} \\ -a_{n1} & -a_{n2} & -a_{n3} & \cdot & \boxed{1-a_{nn}} \end{array}$$

Therefore, the previous constraint implies that

$$\sum_j e_{j_1 j_2 \dots j_n} a_{1j_1} a_{2j_2} \dots a_{nj_n} > 0$$

where $j_1 j_2 \dots j_n$ is a permutation of the integers from 1 to n .²

Usually the values of the principal minors need not be calculated since

¹R. Solow, "On the Structure of Linear Models," Econometrica, XX, (1952), 29-46.

²F. E. Hohn, Elementary Matrix Algebra (New York: Macmillan Co., 1958), p. 31.

the columns of $\left| a_{ij} \right|$ add up to less than one, which is a sufficient condition for the nonsingularity of $(I-a)^{-1}$.

In the regional model considered in Chapter 4, \underline{a} represents the transactions matrix which does not include the rows corresponding to the exogenous sector. Therefore,

$$\sum a_{ij} < 1 \quad (j=1,2 \dots n)$$

$$\sum a_{ij} = 0 \quad (i, j=1,2, \dots n).$$

Therefore, the matrix \underline{a} satisfies the condition

$$\sum \left| a_{ij} \right| = s_j < 1 \quad (j=1,2 \dots n)$$

which then means that $(I-a)^{-1}$ is nonsingular.

Treatment of the Interindustry Model as a Stochastic Equation System

Early in the writing of this thesis, the author developed an interest in the change in the activity coefficients through time. One of the basic restrictive assumptions of interindustry analysis is that the activity coefficients are constant. Presently, the only practical method of overcoming this restriction is to construct a new table of activity coefficients. As previously explained, this involves the use of a tremendous amount of resources. Even with the recent advances in electronic data processing, the accumulating and the processing of the data necessary for the construction of the input-output table are a vast undertaking. The national government has the resources available

for such a study. A comparable study for a region, similar to the one described in the previous chapters also demands large amounts of resources. Unfortunately, the sponsors of a regional study do not have a vast reservoir of resources; this condition leads to the use of national coefficients for regional analyses.

Conceivably, a region could finance one interindustry study. However, eventually it will be outdated since the coefficients change with the passage of time. Furthermore, both the national and regional coefficients have a restriction on them when they are used to predict levels of output and employment for the year 1970, given the bill of goods for that year. The statistical techniques used in forecasting a bill of goods have been developed to provide estimates with a reasonable degree of accuracy. However, regardless of the accuracy in these forecasts, the predicted levels of output and employment generated by the model do not show a comparable degree of accuracy, the reason being that the activity coefficients in 1970 will be different from those used in the current model (1963).

The previous discussion illustrates the need for forecasting changes in the activity coefficients. Even the government, with large quantities of financial resources, cannot construct a table of activity coefficients X years in advance of the current year with complete accuracy. Therefore, there is a definite need for relating activity coefficients to the time factor.

A worthwhile formulation of the interindustry model as a system of simultaneous stochastic equations should involve an expression of

the commodity flow parameters a_{ij} as functions of certain variables measurable in time, which reflect the reasons for change in the aggregate output. Shephard has suggested the use of time-series analysis as a partial solution to the problem.³ In this approach, historical data are collected regarding the output and the final demand of each industry for the past n years. From these data the activity coefficients are calculated and then projected for future years. The first and obvious disadvantage in this approach is that the collection of the data is a formidable task.

In addition to the data problem, there are other statistical problems. As previously stated each a_{ij} is going to be expressed as a function of certain variables, measurable in time, which reflect the reasons for change in the aggregate interindustry flows per unit of aggregate output. Therefore, the functional relation between a_{ij} and its corresponding variables are assumed to be invariant. Hence another problem which must be solved is the choice of the change-explaining-variables. After the introduction of these variables, the previously described time series analysis may be utilized. One additional problem must be considered when the model is treated as a stochastic equation system, i.e., the computation of the maximum likelihood estimates of the parameters used to relate each a_{ij} to its corresponding variable in the time series.

Most economic variables are stochastic (random) in that they are represented by probability distributions. The variables are

³R. W. Shephard, A Survey of Input-Output Research, Rand Corporation Report No. P-309 (Santa Monica, Calif.: Rand Corporation July, 1952).

stochastic since their value is a function of some non-observable phenomena classified as a random error. In addition to random errors, economic data are also distorted by random shocks, i.e., external occurrences which affect a given variable and its corresponding random error. The relationship of a stochastic equation system to the present problem is now considered. By the use of simultaneous equations, a relationship can be made between the observed data, the random errors and the random shocks. A quantitative expression of these relationships is called a stochastic equation system.

As Marschak states, "The role of simultaneous equations is familiar to economic theorists. But it has often been forgotten by economic statisticians who tried to estimate a single stochastic relation as if no other such relations had taken part in determining the observed values of the variables. On the other hand, economic theorists are apt to forget that the observed economic variables are, in general, stochastic. To be susceptible of empirical tests an economic hypothesis must be formulated as a statistical one, i.e., be specified in terms of probability distributions."⁴ It may be stated in conclusion that each economic observation is an outgrowth of a system of complex relations which are simultaneous, stochastic, and dynamic.

Marschak has given a rigorous definition to stochastic models.⁵ The following discussion represents a synthesis of Marschak's work with

⁴T. C. Koopmans (ed.), Statistical Inference in Dynamic Economic Models (New York: John Wiley, 1950), p. 19.

⁵Ibid., p. 3.

recent advances in mathematical statistics. The objective of this discussion is to point out the role that the previously discussed parameters have in the formulation of a model.

A stochastic model M is the a priori information on a system of equations

$$(A.1) \quad \emptyset_g(x, w; a_{(g)}) = 0 \quad g=1, \dots, G$$

and on the joint distribution density function

$$(A.2) \quad f(w; e)$$

where

a priori information represents statements which are obtained independently from known observable data.

\emptyset_g = a scalar element of the vector \emptyset

$a_{(g)}$ = a parameter vector (subvector)

x = (x_1, \dots, x_n) the vector of observable variables

w = (w_1, \dots, w_j) the vector of nonobservable random disturbances

e = a parameter vector

Substituting w from equation A.1 into equation A.2 yields the distribution density function of the observable vector x , i.e., $g_x(x)$. Therefore, the analysis presupposes a knowledge of \emptyset , f , and their parameters a and e . The parameters are usually obtained by using maximum likelihood techniques. Koopmans provides an excellent coverage of this subject.⁶ The activity coefficients in the interindustry model would correspond to the parameter vector $a_{(g)}$ and is subjected to the same statistical techniques as the parameters a and e . The entire structural relation for the system would be given by $X = (I-A)^{-1}Y$.

⁶Ibid., p. 26.

Linear Programming and Factor Substitution in Open Leontief Models

There are many important economic assumptions underlying the construction of the Leontief model. The assumption regarding factor substitution is closely related to the linear programming interpretation of Leontief's model and is the topic of discussion in this section.

The assumption under consideration is concerned with the nature of the production process used by each industry. Leontief assumes each industry produces only one commodity and this production is achieved by only one production process. This assumption has many implications. For example, if labor demands wages which exceed its productivity, there is a tendency for capital to be substituted for labor. One may erroneously conclude that the factor substitution is impossible in the Leontief model as it is in the Walrasian model. An elementary answer to this question is to recall that the mix of factors used in the production of any commodity is obtained after a consideration of all possible processes. Therefore, the problem of substituting one process or activity instead of another is solved before the model is constructed.

Samuelson has shown that a Leontief system with one primary factor is compatible with the general case of substitutability. "With labor the only primary factor, all desirable substitutions have already been made by the competitive market, and no variation in the composition of final output or in the total quantity of labor will give rise to price change in substitution."⁷

⁷T. C. Koopmans (ed), Activity Analysis of Production and Allocation. (New York: John Wiley, 1951), p. 143.

Although Samuelson's Substitution Theorem has solved the theoretical problem regarding factor substitution in Leontief models, there still exists a serious empirical problem. The variables in the model are not homogeneous economic quantities. The activity coefficients are obtained by aggregating the commodity flows of many industries which in turn destroy the complete homogeneity in the data assumed by Samuelson.

The previous discussion concludes that as long as the model takes the process used for the production of any commodity as given, i.e., if the decision making factor is ignored, the problem of substitution will not arise. With the recent advances in the theory of linear programming, the problem of decision making in Leontief models may be expanded.

For illustrative purposes, consider an economy with three industries whose outputs are X_1 , X_2 , and X_3 and whose corresponding final demands are Y_1 , Y_2 , and Y_3 . Let x_{ij} equal money flows from industry i to industry j and $i = 0$ for labor. The problem is to choose that production process which has the minimum labor requirements for a given bill of goods, and may be stated in the following manner:

$$\text{Minimize: } A_{01}X_1 + A_{02}X_2 + A_{03}X_3$$

subject to the following conditions:

$$\begin{array}{rrrrr} (1-a_{11})X_1 & -a_{12}X_2 & -a_{13}X_3 & \geq & Y_1 \\ -a_{21}X_1 & (1-a_{22})X_2 & -a_{23}X_3 & \geq & Y_2 \\ -a_{31}X_1 & -a_{32}X_2 & (1-a_{33})X_3 & \geq & Y_3 \end{array}$$

The solution to the above linear programming problem is not considered in this discussion since there are several excellent books on the subject. However, the example does illustrate the manner in

which linear programming techniques can be applied to Leontief models. The linear programming interpretation of the model provides an evaluative criterion for selecting a specific production process. After the decision is made, assuming a homogeneous product group, Samuelson's Substitution Theorem answers the question regarding the effect of factor substitution on output.

As previously stated, there are three quantitative tools used in interindustry analysis, namely; input-output models, linear programming techniques, and economic activity analysis. The historical development of these approaches is in the above order. Also, the empirical utility of the approaches is in the same order, i.e., input-output analysis is a more valuable tool for an empirical analysis than either linear programming or activity analysis.

Essentially, activity analysis involves an extension of linear programming techniques to general equilibrium theory in economics. Activities are combinations of commodities, some being inputs used in fixed proportions to produce others as outputs in fixed proportions. "An activity is a quantitative concept; it has a level and hence a unit size."⁸

Activity analysis broadens the scope of classical economic analysis. No longer is the model limited to continuous differentiable production functions, i.e., different activities may produce goods in discrete patterns. Activity analysis also recognizes the fact that a firm may produce a variety of products from its production facilities.

⁸ R. G. D. Allen, Mathematical Economics. 2nd ed. revised (London: Macmillan & Co., 1959), p. 568.

Presently, activity analysis is in a formative stage and is not prone to empirical use. However, it is a recent innovation in economic analysis and therefore its present limitations should not discourage additional inquiry into its potential uses.

Dynamic Interpretations of Interindustry Models

The basic model described in Chapter 2 is static since there is no equation in the system which is a function of time. "A system is dynamic if its behavior over time is determined by functional equations in which variables at different points of time are involved in an essential way."⁹

The static model emphasizes the simultaneous flows of goods and services among the producing and consuming sectors of the economy. However, the static model does not consider the effects of the past on the future. The recognition of the importance of time on economic data is a unique addition to the static Leontief Model. The primary assumption of the static model, i.e., that each industry produces one commodity by the use of a fixed technological process, is retained in the dynamic model.

The controversy concerning the merits of static and dynamic models in economics is a lengthy one. Some economists believe that the static solution is the long-run equilibrium solution while others state that economic observations are always changing and therefore must be explained by the use of dynamic models. Given the previous definitions of static and dynamic systems, the two approaches may be reconciled.

⁹P. A. Samuelson, Foundations of Economic Analysis (Cambridge, Mass.: Harvard University Press, 1947), p. 314

Jevons, who had an excellent knowledge of the physical sciences, suggested the following reconciliation of the two approaches. He believed that static models would describe the nature of the forces which lead to an equilibrium level while dynamic models would provide an explanation of the changes which lead to this equilibrium level. Many writers have used this analogy to form the foundation for the following argument. Their belief is that a static model pinpoints important structural relationships whose modification leads to a dynamic model.

Other writers disagree and state that dynamic models must be constructed from principles entirely different from those found in traditional economic theory. Kuznets states that

it is true that the element of empirical generalization and analysis contained in traditional economic theory is of some value for the purpose of explaining problems of change. The singling out of the most important groups of social phenomena, the attempt to show their interconnection, the broad generalization about human behavior and the interaction of individuals, the groups of conditioning factors singled out, all these parts of a static scheme are usually derived from observation of economic life and thus likely to yield results of some value. But the static scheme in its entirety, in the essence of its approach, is neither a basis nor a stepping stone towards a proper treatment of dynamic problems. It may provide some clues for dynamics, but its list of factors is incomplete, its emphasis is misleading, and its essential analytical part, its principle of organization must be discarded if the otherwise difficult problem of economic change is not to be made deceptively simple in the short run and all the more difficult of handling in the long run.¹⁰

The dynamic extension of this interindustry model utilizes the

¹⁰S. Kuznets, Economic Change (New York: W. W. Norton, 1953), p. 41.

methodology suggested by Jevons, i.e., the basic structure found in the static model is expanded upon in order to lead to the construction of a dynamic model. The author does not claim this to be the best or only method for the construction of the dynamic model. These followers of Kuznets are perfectly free to develop a new body of interindustry theory to explain the dynamic case. Unfortunately, the present dynamic theory of interindustry models is in its primitive stage and will probably remain in this stage if no use is made of previous studies.

A Mathematical Statement of the Open Dynamic Model

The open static model is represented by

$$(A.3) \quad X = (I-A)^{-1}Y$$

This model is not concerned with capital formation such as an increase in machinery, inventory, or investment. The dynamic extension of the model relates capital stock to output. The activity coefficients in the static model relate the input from industry i to the output of industry j . In addition to the activity coefficients, the dynamic model uses capital coefficients, b_{ij} . Thus b_{ij} represents the units of product i held per unit flow of product j . Having defined b_{ij} in this manner, C_{ij} represents the capital formation of commodity i held by industry j .

Therefore, the matrix of activity coefficients is as follows:

$$A = \begin{bmatrix} (1-a_{11}) & -a_{12} & -a_{13} & \dots & -a_{1n} \\ -a_{21} & (1-a_{22}) & -a_{23} & \dots & -a_{2n} \\ -a_{31} & -a_{32} & (1-a_{33}) & \dots & -a_{3n} \\ . & . & . & . & . \\ . & . & . & . & . \\ -a_{n1} & -a_{n2} & -a_{n3} & \dots & (1-a_{nn}) \end{bmatrix}$$

The matrix of capital coefficients is:

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{1n} \\ b_{21} & b_{22} & b_{23} & \dots & b_{2n} \\ b_{31} & b_{32} & b_{33} & \dots & b_{3n} \\ \cdot & \cdot & \cdot & & \cdot \\ b_{n1} & b_{n2} & b_{n3} & \dots & b_{nn} \end{bmatrix}$$

In the dynamic model the activity coefficients are considered to be constant, i.e.,

$$x_{ij} = a_{ij}X_j$$

Likewise, in the dynamic model the capital coefficients, b_{ij} , are considered to be constant. Hence

$$C_{ij} = b_{ij}X_j$$

By assuming that the capital formation function is continuous

$$(A.4) \quad \frac{dC_{ij}}{dt} = \frac{b_{ij}dX_j}{dt}$$

Equation (A.4) states that the change in capital formation is equal to b_{ij} times the change in output.

Equation (A.3) can be written in the following form

$$X_i = a_{ij}X_j + Y_i \quad (i = 1, \dots, n)$$

The dynamic model may be constructed from the static model by adding the capital formation term $\frac{dC_{ij}}{dt}$.¹¹ This term represents the change in capital formation with time. Therefore, the model for the

¹¹Allen, p. 363.

dynamic case becomes

$$(A.5) \quad X_i = \sum_{j=1}^n a_{ij}X_j + Y_i + \sum_{j=1}^n \frac{dC_{ij}}{dt} \quad (i = 1, 2, \dots, n)$$

Substituting equation (A.4) into equation (A.5)

$$X_i = \sum_{j=1}^n a_{ij}X_j + Y_i + \sum_{j=1}^n b_{ij} \frac{dX_j}{dt} \quad (i = 1, 2, \dots, n)$$

$$X_i = \sum_{j=1}^n a_{ij}X_j + \sum_{j=1}^n b_{ij} \frac{dX_j}{dt} + Y_i \quad (i = 1, 2, \dots, n)$$

$$(A.6) \quad X_i = \sum_{j=1}^n \left(a_{ij}X_j + b_{ij} \frac{dX_j}{dt} \right) + Y_i \quad (i = 1, 2, \dots, n)$$

In general, equation (A.6) cannot be solved in its present form since there are two variables, X and Y . Therefore, it is necessary to obtain $Y = f(X)$. There are many possible expressions for relating final demand to gross output. Assume that the final demand changes exponentially at a rate g over time t . Therefore

$$(A.7) \quad Y_i = k_i e^{g_i t} \quad (i = 1, 2, \dots, n)$$

Substituting equation (A.7) into equation (A.6)

$$(A.8) \quad X_i = \sum_{j=1}^n \left(a_{ij}X_j + b_{ij} \frac{dX_j}{dt} \right) + k_i e^{g_i t} \quad (i = 1, 2, \dots, n)$$

Fortunately, equation (A.8) is a non-homogeneous, linear, first order ordinary differential equation. Equation (A.8) is the equation for the dynamic model. It corresponds to $X = (I-A)^{-1}Y$ for the static model if written in matrix form.

Positive Equilibrium and the Stability of Dynamic Systems

As previously discussed, the problem of positive equilibrium is important since it represents a meaningful solution in the static case. Positive equilibrium exists if there are no negative outputs for a given positive bill of goods. Solow¹² has arrived at an interesting conclusion concerning the relation of the static model to the dynamic. He states that a system of linear production functions is capable of producing a positive bill of goods if the corresponding dynamic system is stable. Therefore another important use of the dynamic model is in studying the properties of the static model. In order to examine the possibility of reaching a state of positive equilibrium, a restatement of the dynamic model in terms of difference equations is not mandatory. However, there is an advantage in this approach.

Metzler has shown that $a_{ij} \geq 0$, is a necessary and sufficient condition for the stability of the dynamic system, in the sense that all the characteristic roots of \underline{a} are less than unity in modulus and that $(I - \underline{a})$ has all of its principal minors positive.

The static model

$$(A.9) \quad X_i = a_{ij}X_j + Y_i$$

can be written as a difference equation

$$(A.10) \quad Ix(t+1) - ax(t) = Y$$

Solow shows that (A.9) is the static solution of (A.10)

The solution of (A.10) is

$$x(t) = a^t x(0) + (I + a + a^2 + \dots + a^{t-1})Y$$

¹²R. Solow, "On the Structure of Linear Models, " Econometrica, XX (1952), 29-32.

Therefore if $(I + a + a^2 + \dots + a^{t-1})$ converges to $(I-a)^{-1}$, and a^t goes to the null matrix then stability in (A.10) insures non-negativity in (A.9).

The importance of this result is in its empirical use. The application of the Metzler criterion would furnish similar results but the problem of evaluating the principal minors is very tedious for large equation systems. In lieu of evaluating the principal minors, it is usually easier to examine the stability of the dynamic system and then relate it to the non-negative solutions of the static model.

A.2--Definitions of Industry Groups

Endogenous Sectors

Since the definitions of the industries in the endogenous sector are defined in the Standard Industrial Classification Manual, they are not restated. However, definitions of industries in the exogenous sector are not contained in this manual and are now defined.

Exogenous Sectors

The exogenous sectors are those sectors whose activity coefficients are not expected to remain constant, i.e., the output of the sector is not a linear function of its input. The exogenous sectors constitute the final demand or bill of goods for each producing industry in the model.

In the endogenous sector of the model each row element has a corresponding column element, i.e., there is a symmetry between rows and

columns. In the present model, the endogenous sector is a square matrix with 10 columns and 10 rows; the exogenous sector has six entries.

When the model is placed into operation, it may be advantageous to consolidate some of the input entries in the final demand sector. Therefore, although there would be six columns in the final demand sector, there may be less than six rows corresponding to the six columns. This offers no conceptual problems since several of the rows are merely added together when reporting the data. The model no longer gives a square matrix but this problem is resolved by the fact that the matrix which is inverted is square since it is composed of only the entries in the endogenous sector which are symmetrical as discussed above.

Construction.-- This sector includes all expenditures or receipts arising from the erection of immobile structures and utilities including any additions to service facilities which become an integral part of the operating structure.¹³

Government.-- This sector is concerned with the transactions of the state, local, and federal governments with the remaining sectors in the model. The government row in the model represents tax payments of the non-governmental sectors to the government. These taxes include excise taxes as well as corporate income taxes. The government column

¹³W. E. Martin and H. O. Carter, A California Interindustry Analysis Emphasizing Agriculture, Giannini Foundation Report No. 250, (University of California, February, 1962).

represents purchases of goods and services made by the government from the various industries in the model as well as transfer payments and subsidies.

Inventory Change.-- This sector recognizes the lag between production and consumption. An inventory addition, representing a surplus of production over consumption is entered as a positive number, and inventory depletion representing a surplus of consumption over production is entered as a negative number in the transactions table. Due to the difficulties in obtaining these data as a producing entry, it is recommended that the amount attributed to this element be included in the households row. Therefore, although there will be a column for inventory change, there will be no corresponding row entry.

Gross Private Capital Formation.-- The purchases of capital machinery and other capital goods are recorded in this sector. Items related to capital expenditures which are not elsewhere classified are included in this sector; for example, engineering fees not included in current construction activity. One must note that depreciation is not to be included as a negative amount but is to be distributed to the using sector through the household sector.

Households.-- The household column in the household sector represents personal consumption expenditures for goods and services by private individuals; each column entry represents a purchase or an input. The household row represents payments to individuals; these flows take the form of wages and salaries, depreciation, and interest.

Imports and Exports.-- Conceptually, this is a very important entry in the final demand sector. The addition of this sector is one of the important changes necessary to change a general model to a regional model. Ideally, one should examine detailed data concerning the import and export of goods in and out of the region under consideration, i.e., the Duval County area. Unfortunately, such information is very difficult to obtain, and even with the expenditure of large amounts of resources, it is doubtful whether such information may be found. Consequently the only course of action is to compare the amount of each product used in the county with the amount of each product produced in the county and assume that the difference was either an import or an export. Hence imports are a net relationship and are used as the balancing sector between production and consumption.

BIBLIOGRAPHY

BOOKS

- Alderson, W. Marketing Behavior and Executive Action. Homewood, Ill.,: R. D. Irwin, 1957.
- Alderson, W., and Shapiro, S. J. (ed.). Marketing and the Computer. Englewood Cliffs, N. J.: Prentice-Hall, 1963.
- Allen, R. G. D. Mathematical Economics. 2nd ed. revised. London: Macmillan & Co., 1959.
- Arrow, K. J., Karlin, S., and Suppes, P. (ed.). Mathematical Models in the Social Sciences. Stanford, Calif.: Stanford University Press, 1960.
- Barna, T. (ed.). The Structural Interdependence of the Economy. Proceedings of an International Conference on Input-Output Analysis. New York: John Wiley, 1954.
- Bartels, R. The Development of Marketing Thought. Homewood, Ill.: R. D. Irwin, 1962.
- Beach, E. F. Economic Models. New York: John Wiley, 1957.
- Blodgett, R. H. Our Expanding Economy, New York: Rhinehart & Co., 1955.
- Briefs, H. W. Three Views of Method in Economics. Washington, D. C.: Georgetown University Press, 1960.
- Charlesworth, J. C. Mathematics and the Social Sciences. Philadelphia: The American Academy of Political and Social Sciences, 1963.
- Charnes, A., Cooper, W. W., and Henderson, A. An Introduction to Linear Programming. New York: John Wiley, 1953.
- Charnes, A., and Cooper, W. W. Marketing Models and Industrial Applications of Linear Programming. Vol. I. New York: John Wiley, 1961.
- Chenery, H. B., and Clark, P. G. Interindustry Economics. New York: John Wiley, 1959.

- Churchman, C. W. Elements of Logic and Formal Science. New York: J. B. Lippincott Co., 1940.
- Dorfman, R., Samuelson, P. A., and Solow, R. M. Linear Programming and Economic Analysis. New York: McGraw-Hill, 1958.
- El 'Sgol 'Ts, L. E. Differential Equations. Delhi, India: Hindustan Publishing Corp., 1961.
- Forrester, J. W. Industrial Dynamics. New York: John Wiley, 1961.
- Freudenthal, H. (ed.). The Concept and Role of the Model in Mathematics and the Natural and Social Sciences. New York: Gordon and Breach, 1961.
- Gale, D. The Theory of Linear Economic Models. New York: McGraw-Hill, 1960.
- Glasstone, S. Theoretical Chemistry. New York: D. Van Nostrand, 1961.
- Hadley, G. Linear Algebra. Reading, Mass.: Addison-Wesley, 1961.
- Hadley, G. Linear Programming. Reading, Mass.: Addison-Wesley, 1962.
- Hohn, F. E. Elementary Matrix Algebra. New York: Macmillan Co., 1958.
- Hood, Wm. C., and Koopmans, T. C. Studies in Econometric Method. New York: John Wiley, 1952.
- Isard, W. Methods of Regional Analysis: An Introduction to Regional Science. New York: John Wiley, 1960.
- Karlin, S. Mathematical Methods and Theory in Games Programming and Economics. Vol. I. Reading, Mass: Addison-Wesley, 1959.
- Kelley, E. J., and Lazer, W. Managerial Marketing. Homewood, Ill.: R. D. Irwin, 1958.
- Kemeny, J. G., and Others. Finite Mathematical Structures. Englewood Cliffs, N. J.: Prentice-Hall, 1958.
- Kemeny, J. G., and Snell, J. L. Mathematical Models in the Social Sciences. Englewood Cliffs, N. J.: Prentice-Hall, 1962.
- Knight, F. H. On the History and Method of Economics. Chicago, Ill.: University of Chicago Press, 1956.
- Koopmans, T. C. (ed.). Activity Analysis of Production and Allocation. New York: John Wiley, 1951.

- Koopmans, T. C. (ed.). Statistical Inference in Dynamic Economic Models. New York: John Wiley, 1950.
- Kuznets, S. Economic Change. New York: W. W. Norton, 1953.
- Lazar, W. "An Investigation of Marketing Models," in Marketing: A Maturing Discipline. Edited by M. L. Bell. Chicago: American Marketing Association, 1961, pp. 47-53.
- Lazarsfeld, Paul F. (ed.). Mathematical Thinking in the Social Sciences. Glencoe, Illinois: The Free Press, 1954.
- Leontief, W. The Structure of the American Economy, 1919-1929. Cambridge, Mass.: Harvard University Press, 1951.
- Leontief, W. Studies in the Structure of the American Economy. New York: Oxford University Press, 1953.
- Lerner, D., and Lasswell, H. D. (ed.). The Policy Sciences. Stanford, Calif.: Stanford University Press, 1951.
- Marshall, Alfred. Principles of Economics. 8th ed. New York: Macmillan Co., 1948.
- Mickley, H. S., Sherwood, T. C., and Reed, C. E. Applied Mathematics in Chemical Engineering. New York: McGraw-Hill, 1957.
- Miller, K. S. The Calculus of Finite Differences and Difference Equations. New York: Henry Holt, 1960.
- Morgenstern, O. Economic Activity Analysis. New York: John Wiley, 1954.
- Morgenstern, O. On the Accuracy of Economic Observations. Princeton, N. J.: Princeton University Press, 1950.
- National Bureau of Economic Research. Input-Output Analysis: An Appraisal. Conference on Research and Wealth. Princeton, N.J.: Princeton University Press, 1955.
- Netherlands Economic Institute (ed.). Input-Output Relations - Proceedings of a Conference of Inter-industrial Relations Held at Driebergen, Holland. Kroese, N.V. Leiden: H. E. Stenfert, 1953.
- Papandreou, A. G. Economics as a Science. New York: J. P. Lipponcott Co., 1958.
- Rogin, Leo. The Meaning and Validity of Economic Theory. New York: Harper Brothers, 1956.

- Samuelson, P. A. Foundations of Economic Analysis. Cambridge, Mass.: Harvard University Press, 1947.
- Schoeffler, S. The Failures of Economics: A Diagnostic Study. Cambridge, Mass.: Harvard University Press, 1955.
- Schumpeter, J. A. History of Economic Analysis. New York: Oxford University Press, 1954.
- Schwartz, J. T. Lectures on Mathematical Method in Analytical Economics. New York: Gordon & Breach, 1961.
- Shephard, R. W. Cost and Production Functions. Princeton, New Jersey: Princeton University Press, 1953.
- Sokolnikoff, I. S. Advanced Calculus. New York: McGraw-Hill, 1939.
- Taylor, H. S., and Glasstone, S. A Treatise on Physical Chemistry, Vol. I, Atomistics and Thermodynamics. New York: D. Van Nostrand, 1947.
- Thrall, R. M., Coombs, C. H., and Davis, R. L. Decision Processes. New York: John Wiley, 1960.
- Valavanis, S. Econometrics. New York: McGraw-Hill, 1959.
- Von Mises, L. Epistemological Problems of Economics. Princeton, N. J.: D. Van Nostrand, 1960.
- Walras, Leon. Elements of Pure Economics. Translated by William Jaffe. Homewood, Ill.: R. D. Irwin, 1954.

PERIODICALS

- Altschul, E., and Biser, E. "The Validity of Unique Mathematical Models in Science," Philosophy of Science, XIV and XV (1947 and 1948),, 11-24.
- Ara, K. "The Aggregation Problem in Input-Output Analysis," Econometrica, XXVII (1959), pp. 257-262.
- Beshers, J. M. "Models and Theory of Construction," American Sociological Review, XXII (1957), pp. 32-38.
- Blalock, H. M., Jr., and Blalock, A. G. "Toward a Classification of Systems Analysis in the Social Sciences," Philosophy of Science, XXVI (1959), pp. 84-92.

- Briggs, F. E. A. "On Problems of Estimation on Leontief Models," Econometrica, XXV (1957), pp. 444-455.
- Cameron, B. "The Construction of the Leontief System," The Review of Economic Studies, XIX (1951), pp. 19-27.
- Cameron, B. "The Labor Theory of Value in Leontief Models," The Economic Journal, LXII (1952), pp. 191-197.
- Cameron, B. "The Production Function in Leontief Models," The Review of Economic Studies, XX (1952-1953), pp. 62-69.
- Debreu, G., and Herstein, I. N. "Nonnegative Square Matrices," Econometrica, XXI (1953), pp. 597-607.
- Dorfman, R. "The Nature and Significance of Input-Output," The Review of Economics and Statistics, XXXVI (1954), pp. 121-133.
- Evans, W. D. "The Effect of Structural Matrix Errors on Interindustry Relations Matrix," Econometrica, XXII (1954), pp. 461-479.
- Evans, W. D., and Hoffenberg, M. "The Interindustry Relations Study for 1947," The Review of Economics and Statistics, XXXIV (1952), pp. 97-142.
- Fei, J. D. H. "A Fundamental Theorem for the Aggregation Problem of Input-Output Analysis," Econometrica, XXIV (1956), pp. 400-412.
- Feibleman, J. R. "Mathematics and Its Applications in the Sciences," Philosophy of Science, XXIII (1956), pp. 204-215.
- Fisher, F. M. "On the Analysis of History and Interdependence of the Social Sciences," Philosophy of Science, XXVII (1960), pp. 147-158.
- Fisher, W. D. "Criteria for Aggregation in Input-Output Analysis," The Review of Economics and Statistics, XL (1950), pp. 250-260.
- Gewirth, A. "Can Man Change Laws of Social Science?" Philosophy of Science, XXI (1954), pp. 229-241.
- Gewirth, A. "Subjectivism and Objectivism in the Social Sciences," Philosophy of Science, XXI (1954), pp. 157-163.
- Ghosh, A. "A Note on the Leontief Model with Non-Homogeneous Production Functions," Metroeconomica, XII, No. 1 (1960), pp. 14-21.
- Grunberg, E. "Notes on the Verifiability of Economic Laws," Philosophy of Science, XXIV (1957), pp. 337-348.

- Handy, Rollo. "Philosophy's Neglect of the Social Sciences," Philosophy of Science, XXV (1958), pp. 117-124.
- Harbarger, A. C. "Pitfalls in Mathematical Model-Building," American Economic Review, XLII (1952), pp. 855-865.
- Hatanaka, M. "Note on Consolidation within a Leontief System," Econometrica, XX (1952), pp. 301-303.
- Hindrichs, G. "Toward a Philosophy of Operations Research," Philosophy of Science, XX (1953), pp. 59-66.
- Hurwicz, L. "Input-Output Analysis and Economic Structure," American Economic Review, XLV (1955), pp. 626-636.
- Klein, L. R. "The Interpretation of Professor Leontief's System" The Review of Economic Studies, XXIV (1), No. 63 (1956-1957) pp. 131-136.
- Kuenne, R. E. "Walras, Leontief, and the Interdependence of Economic Activities," The Quarterly Journal of Economics, LXVIII (1954), pp. 323-354.
- Lande, A. "Continuity, A Key to Quantum Mechanics," Philosophy of Science, XX (1953), pp. 101-109.
- Leontief, W. "Input-Output Analysis and Its Use in Peace and War Economies," American Economic Review, XXXIX (1949), pp. 211-225.
- Levey, M. J., Jr. "Some Basic Methodological Difficulties in Social Science," Philosophy of Science, XVII (1950), pp. 287-301.
- McKenzie, L. "An Elementary Analysis of the Leontief Systems," Econometrica, XXV, No. 3 (1957), pp. 436-462.
- Meadows, P. "Models, Systems and Science," American Sociological Review, XXII, No. 1 (1957), p. 3.
- Meyer, H. "On the Heuristic Value of Scientific Models," Philosophy of Science, XVIII (1951), p. 114.
- Moore, F. T., and Petersen, J. W. "Regional Analysis: An Interindustry Model of Utah," The Review of Economics and Statistics, XXXVII (1955) pp. 368-381.
- Nicosia, F. M. "Marketing and Alderson's Functionalism," Journal of Business, XXV (1962), pp. 403-412.
- Phillips, A. "The Variation of Technical Coefficients in the Anti-Friction Bearing Industry," Econometrica XXIII (1955) pp. 432-441.

- Rudner, R. "The Scientist Qua Scientist Makes Value Judgments," Philosophy of Science, XX (1953), pp. 1-6.
- Ryan, J. M. "The Leontief System," Southern Economic Journal (1953) pp. 481-493.
- Solow, R. "On the Structure of Linear Models," Econometrica, XX, No. 1 (1952), pp. 29-46.
- Stephenson, W. "Postulates of Behaviorism," Philosophy of Science, XX (1953), pp. 110-120.
- Theil, H. "Linear Aggregation in Input-Output Analysis," Econometrica XXV (1957), pp. 111-122.
- Wurtele, Z. S. "A Note on Some Stability Properties of Leontief's Dynamic Models," Econometrica, XXVII, No. 4 (1959), pp. 672-675.

REPORTS

- American Management Association. Simulation and Gaming: A Symposium. (Report No. 55) New York: The American Management Association, 1961.
- Bowlen, E., and Heady, E. O. Optimum Combinations of Competitive Crops. (Iowa State College Research Bulletin No. 426.) Ames, Iowa: Agriculture Experiment Station, Iowa State College, April 4, 1955.
- Carter, H. O., and Heady, E. O. An Input-Output Analysis Emphasizing Regional and Commodity Sectors of Agriculture. (Iowa State College Research Bulletin No. 469.) Ames, Iowa: Agriculture and Home Economics Experiment Station, Iowa State College, September, 1959.
- Hatanaka, M. Testing the Workability of Input-Output Analysis. Economic Research Report. Princeton, N. J.: Princeton University, April 22, 1957.
- Martin, W. E. and Carter, H. O. A California Interindustry Analysis Emphasizing Agriculture. (Giannini Foundation Report No. 250) Berkeley, California: California Agriculture Experiment Station, University of California, February, 1962.

Schnittker, J. A., and Heady, E. O. Application of Input-Output Analysis to a Regional Model Stressing Agriculture. (Iowa State College Research Bulletin No. 454) Ames, Iowa: Agricultural and Home Economics Experimental Station, Iowa State College, December, 1957.

Shephard, R. W. A Survey of Input-Output Research. (Rand Corporation Report No. P-309) Santa Monica, California: Rand Corporation, July, 1952.

Tiebout, C. M. The Community Economic Base Study. (Supplementary Paper No. 16) New York: Committee for Economic Development, December, 1962.

PUBLIC DOCUMENTS

Executive Office of the President, Bureau of Budget. Standard Industrial Classification Manual: 1957.

U. S. Department of Commerce, Bureau of the Budget. U. S. Census of Population: 1960. Florida. PC (1) - 11-A: Number of Inhabitants.

U. S. Department of Commerce, Bureau of the Budget. U. S. Census of Population: 1960, Florida. PC (1) - 11-B: General Population Characteristics.

U. S. Department of Commerce, Bureau of the Budget. U. S. Census of Population: 1960, Florida. PC (1) - 11-C: General Social and Economic Characteristics.

U. S. Department of Commerce, Bureau of the Budget. U. S. Census of Population: 1960, Florida. PC (1) - 11-D: Detailed Characteristics.

U. S. Department of Commerce, Bureau of the Census. 1958 Census of Manufacturers: 1961. Area Statistics, Vol. III.

U. S. Department of Commerce, Bureau of the Census. 1957 Census of Governments: 1959. Government in Florida, Vol. VI, No. 8.

U. S. Department of Commerce, Bureau of the Census. Historical Statistics of the United States: 1960.

U. S. Department of Commerce. Governmental Finances in 1958: 1959.

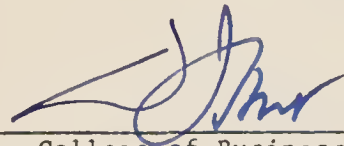
- U. S. Department of Commerce, Bureau of the Census. Statistical Abstract of the United States: 1962.
- U. S. Department of Commerce, Bureau of the Census. U. S. Summary, U. S. Census of Population: 1950.
- U. S. Department of Labor, Bureau of Labor Statistics. Survey of Consumer Expenditures 1960-1961. Advance Report of Consumer Expenditures and Income: 1963.
- Wharton School of Finance and Commerce. Study of Consumer Expenditures, Incomes, and Savings. Vol. XVIII, Summary of Family Incomes, Expenditures, and Savings All Urban Areas Combined - 1950:
U. S. Department of Labor, Bureau of Labor Statistics, 1957.

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This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Business Administration and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

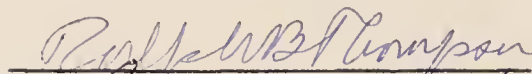
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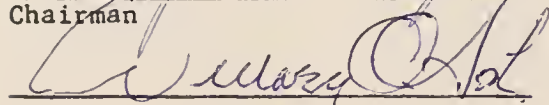
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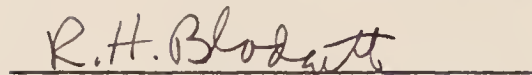
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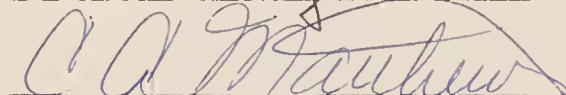
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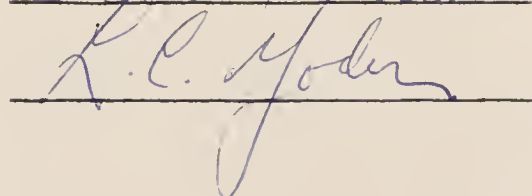


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